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Mathematical Modeling And Behavior Analysis Of A Refrigerating Unit in Milk Plant

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Abstract

In this paper we describe a Refrigerating Unit in Milk plant which comprises of four subsystems. The Refrigerating Unit in Milk plant basically comprises of four subsystems specifically Compressor, Condenser, Expansion Device and Evaporator. A single repairman who examines and repairs the units as and when need emerge. Availability of Refrigerating Unit in Milk plant is determined with the assistance of RPGT and accessibility of the arrangement of Refrigerating Unit in Milk plant for various values of repair and failure rates of subsystems is additionally determined. Profit optimization is also examined. System behavior is discussed with the help of graphs and tables.

Keywords:- Availability, Base-State, MTSF, Steady State.

1. Introduction:

In this paper the importance of Markov process is shown by analyzing the reliability function and availability of Refrigeration Plant arranged in Rohtak District. A refrigeration unit involves of four main mechanisms namely Compressor, Condenser, Expansion Device and Evaporator. Refrigeration plants are thought to be only viable when all four units are in good operating condition. When each of the four units is in good operating condition, the system operates at maximum efficiency. When three out of four units are in good operating condition, it operates at a decreased capacity. When two or more units flop, the system is in a failed condition. There are separate continuous failure and repair rates for all four units. A single repairman is available 24*7. The state transition diagram representing the system states is presented as it gives availability function and the updated diagram leads to the appearance for reliability function. While applying the normalizing condition, the expression for steady state availability is obtained and optimized by using genetic algorithm. The purpose of this study is to obtain (1) reliability function and availability function both from Markov diagram of Refrigeration Plant arranged in Rohtak District an expression of steady state availability optimization model with components' constant failure and repair rates. Kumar, A. and Garg, D.[2019] have discussed the reliability technology theory and its applications. Kumar, A., Goel, P. and Garg, D. [2018] have studied behaviour analysis of a bread making system. Kumar, A., et. al. [2019] analyzed sensitivity analysis of a cold standby framework with priority for preventive maintenance consist two identical units with server failure utilizing RPGT. Present paper consists two units one of which is online while other is kept is cold standby mode. Online & cold standby unit are indistinguishable in nature & have just two modes one is good and other is totally failed. Rajbala, et al. [2019] have studied the system modeling and analysis: a case study EAEP manufacturing plant. Kumar, A., Goel, P., Garg, D., and Sahu, A. [2017] have studied behavior analysis in the urea fertilizer industry. Kumar, A., Garg, D., and Goel, P. [2017] have examined the mathematical modeling & profit analysis of an edible oil refinery plant. Kumar, A., Garg, D., and Goel, P. [2019] studied mathematical modeling & behavioral analysis of a washing unit in paper mill. Kumar, A et. al. [2018] paper analyzed sensitivity analysis of 3:4:: good system plant. This paper further presents the time dependent and consistent state accessibility when failure and repair rates are variable and steady individually. The mathematical problem along these lines created has additionally been solved systematically and numerically for few decisions of the failure/repair rates of subsystems. A solitary worker who inspects and repairs the units as and when need emerge. Fuzzy concept is utilized to decide disappointment conditions of a unit. Assuming the worker report, that unit isn't repairable; it is supplanted by another one. Taking failure rates dramatic, repair rates general and contemplating different probabilities, a transition diagram of system is created to decide Primary, Secondary & Tertiary circuits and Base state. Problem is defined and solved utilized RPGT. Repair/Failure is statistically independent. Expressions for system parameters for example MTSF, availability, number of server visits and server of busy period are assessed to consider the behavior of the framework

3. Assumptions and Notations

1. Switching over is imperfect.
2. A single repairman is available 24*7.

λ_1 : Constant failure rate of the unit 'E' from full capacity to complete failure.

λ_2: Constant failure rate of the standby 'C'; λ_3: Constant failure rate of the unit 'H'.

θ_1: Constant repair rate of main unit; θ_2: Constant repair rate of standby.

θ_3: Constant repair rate of unit 'H'; θ_4: Constant repair rate of the switch.

E/e: Unit 'A' in good / failed state.

C/ (C)//c: Redundant unit in operative/ standby / failed state.

H / h : Unit 'C' in operative / failed state

S/s: Switch in operative/ failed state.

4.Transition Diagram of the System:

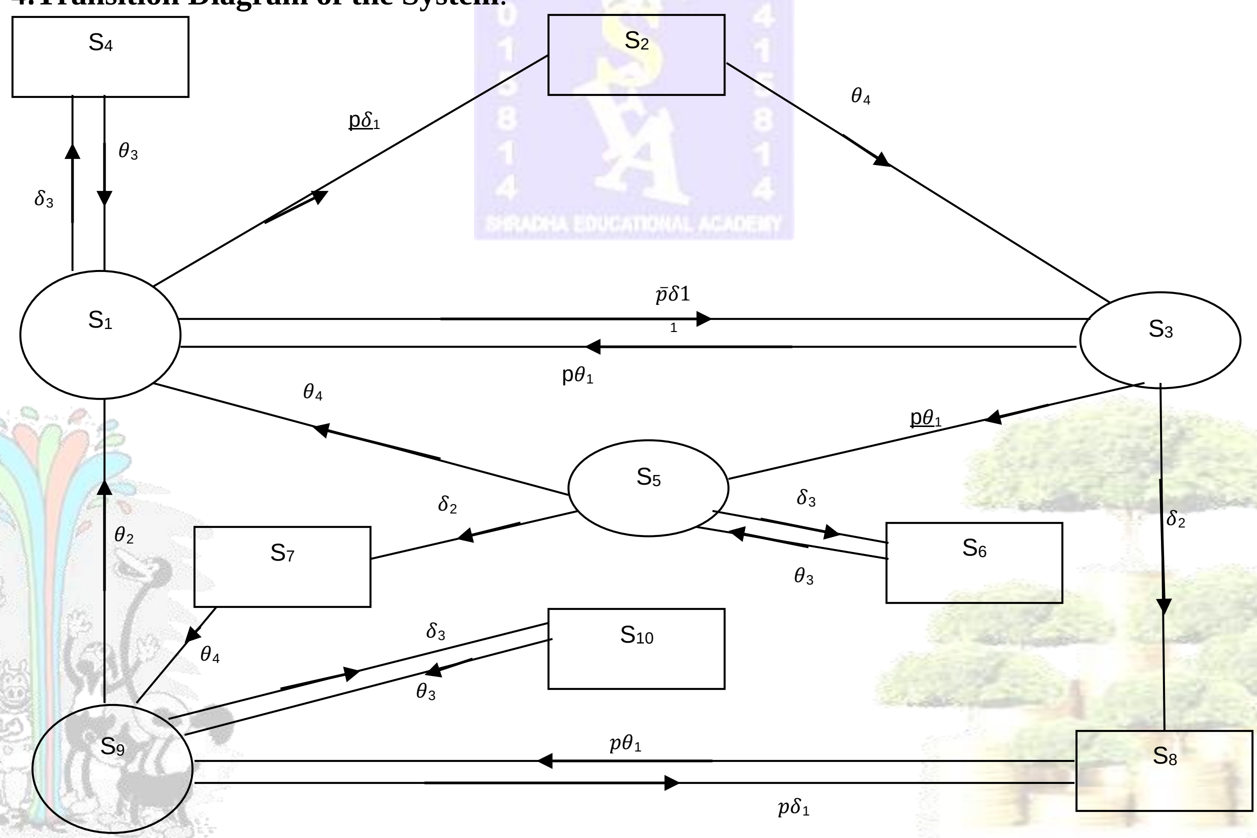

Figure 1: Transition Diagrams

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S_1 = E(C)HS; S_2 = e(C)Hs; S_3 = e(C)HS; S_4 = E(C)hS; S_5 =E(C)Hs
S_6 = e(C)hS; S_7 = EcHs; S_8 = ecHS; S_9 = EcHS; S_{10} = EchS

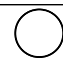
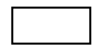
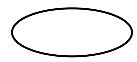
State	Symbol
Up-state	
Failed State	
Reduced State	

Table 1

5. Transition Probabilities and Mean Sojourn Times:

Table: 2 Transition Probabilities

$q_{i,j}(t)$	$p_{i,j}=q_{i,j}^*(0)$
$q_{1,2}= p\delta_1 e^{-(p\delta_1+\bar{p}\delta_1+\delta_3)t}$ $q_{1,3}= \bar{p}\delta_1 e^{-(p\delta_1+\bar{p}\delta_1+\delta_3)t}$ $q_{1,4} = \delta_3 e^{-(p\delta_1+\bar{p}\delta_1+\delta_3)t}$	$p_{1,2}= p\delta_1/(p\delta_1 + \bar{p}\delta_1 + \delta_3)$ $p_{1,3}= \bar{p}\delta_1/(p\delta_1 + \bar{p}\delta_1 + \delta_3)$ $p_{1,4}= \delta_3/(p\delta_1 + \bar{p}\delta_1 + \delta_3)$
$q_{2,1} = \bar{p}\delta_1 e^{-(\bar{p}\delta_1+\theta_4)t}$ $q_{2,3} = \theta_4 e^{-(\bar{p}\delta_1+\theta_4)t}$	$p_{2,1}= \bar{p}\delta_1/(\bar{p}\delta_1 + \theta_4)$ $p_{2,3}= \theta_4/(\bar{p}\delta_1 + \theta_4)$
$q_{3,1}= p\theta_1 e^{-(p\theta_1+\bar{p}\theta_1+\delta_1)t}$ $q_{3,5}= \bar{p}\theta_1 e^{-(p\theta_1+\bar{p}\theta_1+\delta_1)t}$ $q_{3,8} = \delta_2 e^{-(p\theta_1+\bar{p}\theta_1+\delta_2)t}$	$p_{3,1}= p\theta_1/(p\theta_1 + \bar{p}\theta_1 + \delta_2)$ $p_{3,5}= \bar{p}\theta_1/(p\theta_1 + \bar{p}\theta_1 + \delta_2)$ $p_{3,8}= \delta_2/(p\theta_1 + \bar{p}\theta_1 + \delta_2)$
$q_{4,1} = \theta_3 e^{-(\theta_3)t}$	$p_{4,1}= 1$
$q_{5,1}= \theta_4 e^{-(\theta_4+\delta_3+\delta_2)t}$ $q_{5,6}= \delta_3 e^{-(\theta_4+\delta_3+\delta_2)t}$ $q_{5,7} = \delta_2 e^{-(\theta_4+\delta_3+\delta_2)t}$	$p_{5,1}= \theta_4/(\theta_4 + \delta_3 + \delta_2)$ $p_{5,6}= \delta_1/(\theta_4 + \delta_3 + \delta_2)$ $p_{5,7}= \delta_2/(\theta_4 + \delta_3 + \delta_2)$
$q_{6,5} = \theta_3 e^{-(\theta_3)t}$	$p_{6,5}= 1$
$q_{7,9} = \theta_4 e^{-(\theta_4)t}$	$p_{7,9}= 1$
$q_{8,9} = p\delta_3 e^{-(p\delta_3)t}$	$p_{8,9}= 1$
$q_{9,1}= \theta_2 e^{-(\theta_2+p\delta_1+\delta_3)t}$ $q_{9,8}= p\delta_1 e^{-(\theta_2+p\delta_1+\delta_3)t}$ $q_{9,10} = \delta_3 e^{-(\theta_2+p\delta_1+\delta_3)t}$	$p_{9,1}= \theta_2/(\theta_2 + p\delta_1 + \delta_3)$ $p_{9,8}= p\delta_1/(\theta_2 + p\delta_1 + \delta_3)$ $p_{9,10}= \delta_3/(\theta_2 + p\delta_1 + \delta_3)$
$q_{10,9} = \theta_3 e^{-(\theta_3)t}$	$p_{10,9}= 1$

Table: 3 Mean Sojourn Times

$R_i(t)$	$\mu_i=R_i^*(0)$
$R_1(t) = e^{-(p\delta_1+\bar{p}\delta_1+\delta_3)t}$	$\mu_1=1/(p\delta_1 + \bar{p}\delta_1 + \delta_3)$
$R_2(t) = e^{-(\bar{p}\delta_1+\theta_4)t}$	$\mu_2=1/(\bar{p}\delta_1 + \theta_4)$
$R_3(t) = e^{-(p\theta_1+\bar{p}\theta_1+\delta_2)t}$	$\mu_3=1/(p\theta_1 + \bar{p}\theta_1 + \delta_2)$

$R_4(t) = e^{-(\theta_3)t}$	$\mu_4=1/(\theta_3)$
$R_5(t) = e^{-(\theta_4+\delta_3+\delta_2)t}$	$\mu_5=1/(\theta_4+\delta_3+\delta_2)$
$R_6(t) = e^{-(\theta_3)t}$	$\mu_6=1/(\theta_3)$
$R_7(t) = e^{-(\theta_4)t}$	$\mu_7=1/(\theta_4)$
$R_8(t) = e^{-(p\delta_3)t}$	$\mu_8=1/(p\delta_3)$
$R_9(t) = e^{-(\theta_2+p\delta_1+\delta_3)t}$	$\mu_9=1/(\theta_2+p\delta_1+\delta_3)$
$R_{10}(t) = e^{-(\theta_3)t}$	$\mu_{10}=1/(\theta_3)$

6. Transition Probability Factors: - The mean time to system failure and all the key parameters of the system (under steady state conditions) are evaluated by using Regenerative Point Graphical Technique (RPGT) and using '0' as the base state of the system as under: -

$V_{1,1}$= 1 (Verified)

$V_{1,2}$ = (1, 2) = $p_{1,2}$

$V_{1,3}$= …… continued

7. Evaluation of Parameters: - The MTSF and all key parameters of the framework are estimated by applying RPGT taking '0' as base state.

(a) MTSF (T_0): The regenerative un-failed states to which the framework can transit (Initial state '1') before entering any failed state are: for 'ξ' = '1',

$$\text{MTSF }(T_0) = \left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r(s_{ff})} i\right)\right\}\mu_i}{\prod_{m_1\neq\xi}\{1-V_{\overline{m_1,m_1}}\}}\right\}\right] \div \left[1-\sum_{s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r(s_{ff})} \xi\right)\right\}\mu_i^1}{\prod_{m_2\neq\xi}(1-V_{\overline{m_2,m_2}})}\right\}\right]$$

$= [(1, 1)\mu_1 + (1, 2)\mu_2 + (1, 2, 3)\mu_3 + (1, 3)\mu_4 + (1, 3, 5)\mu_5 + (1, 2, 3, 5)\mu_6]/$
$[1 - (1, 3, 1) + (1, 2, 3, 1) + (1, 3, 5, 1) + (1, 2, 3, 5, 1)$

$= [\mu_1 + p_{1,2}\mu_2 + p_{1,2}p_{2,3}\mu_3 + p_{1,3}\mu_4 + p_{1,3}p_{3,5}\mu_5 + p_{1,2}p_{2,3}p_{3,5}\mu_6]/$
$[1-(p_{1,3}p_{3,1}) + (p_{1,2}p_{2,3}p_{3,1}) + (p_{1,3}p_{3,5}p_{5,1}) + (p_{1,2}p_{2,3}p_{3,5}p_{5,1})]$

(b) Availability of the System (A_0): The regenerative states at which the framework is available are j=1, 3, 5, 9 and regenerative states are i= 1 to 10 for 'ξ'='1',

$$A_0 = \left[\sum_{j,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r} j\right)\right\}f_j\mu_j}{\prod_{m_1\neq\xi}\{1-V_{m_1,m_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r} i\right)\right\}}{\prod_{m_2\neq\xi}(1-V_{m_2,m_2})}\right\}\right]$$

$A_0 = \left[\sum_j V_{\xi,j}.f_j.\mu_j\right] \div \left[\sum_i V_{\xi,i}.\mu_i^1\right]$

$=[V_{1,1}f_1\mu_1 + V_{1,3}f_3\mu_3 + V_{1,5}f_5\mu_5 + V_{1,9}f_9\mu_9] \div [V_{1,3}\mu_3^1 + V_{1,1}\mu_1^1 + V_{1,2}\mu_2^1 + V_{1,4}\mu_4^1$
$+ V_{1,5}\mu_5^1 + V_{1,6}\mu_6^1 + V_{1,7}\mu_7^1 + V_{1,8}\mu_8^1 + V_{1,9}\mu_9^1 + V_{1,10}\mu_{10}^1]$

$=[V_{1,1}\mu_1 + V_{1,3}\mu_3 + V_{1,5}\mu_5 + V_{1,9}\mu_9] \div D$

$D= [V_{1,1}\mu_1+V_{1,2}\mu_2+V_{1,3}\mu_3+V_{1,4}\mu_4 + V_{1,5}\mu_5 + V_{1,6}\mu_6 + V_{1,7}\mu_7 + V_{1,8}\mu_8 + V_{1,9}\mu_9 + V_{0,10}\mu_{10}]$

(c) Busy Period of the Server (B_0): The regenerative states where the server is busy while doing repairs are j = 2 to 10

$$B_0 = \left[\sum_{j,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r} j\right)\right\}n_j}{\prod_{m_1\neq\xi}\{1-V_{m_1,m_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\left\{pr\left(\xi \xrightarrow{s_r} i\right)\right\}\mu_i^1}{\prod_{m_2\neq\xi}(1-V_{m_2,m_2})}\right\}\right]$$

$B_0 = (V_{1,2}n_2+V_{1,3}n_3+V_{1,4}n_4+V_{1,5}n_5+V_{1,6}n_6+V_{1,7}n_7+V_{1,8}n_8+V_{1,9}n_9+ V_{1,10}n_{10})/D$

$D= [V_{1,1}\mu_1+V_{1,2}\mu_2+V_{1,3}\mu_3+V_{1,4}\mu_4 + V_{1,5}\mu_5 + V_{1,6}\mu_6 + V_{1,7}\mu_7 + V_{1,8}\mu_8 + V_{1,9}\mu_9 + V_{0,10}\mu_{10}]$

(d) Expected number of server's visits (V_0): The regenerative state to which server visits a fresh are j=2, 3, 4 and the regenerative states are j=0 to 10.

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$$V_0 = \left[\sum_{j,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} j)\}}{\prod_{k_1\neq\xi}\{1-V_{k_1,k_1}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi \xrightarrow{s_r} i)\}\mu_i^1}{\prod_{k_2\neq\xi}(1-V_{k_2,k_2})}\right\}\right]$$

$V_0 = (V_{1,2} + V_{1,3} + V_{1,4})/D$

$D = [V_{1,1}\mu_1 + V_{1,2}\mu_2 + V_{1,3}\mu_3 + V_{1,4}\mu_4 + V_{1,5}\mu_5 + V_{1,6}\mu_6 + V_{1,7}\mu_7 + V_{1,8}\mu_8 + V_{1,9}\mu_9 + V_{0,10}\mu_{10}]$

8. **Particular Cases**: -

For $\delta_i = \delta\,; 1 \le i \le 3;\ \theta_i = \theta;\ 1 \le i \le 4;$ p = 1, p = 0,

9. Analytical Discussion:

Mean Time to System Failure (T_0):-

Table 4: MTSF

δ \ θ	0.50	0.60	0.70
0.10	2.43	2.30	2.34
0.20	2.23	2.17	2.12
0.30	2.04	1.92	1.81

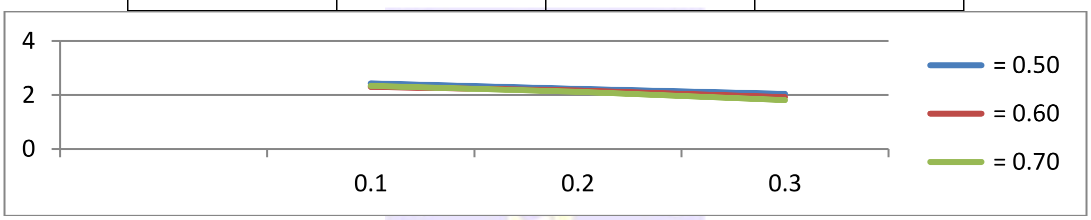

Figure 2: MTSF

Availability of the System (A_0):-

Table 5: Availability of System

δ \ θ	0.50	0.60	0.70
0.10	0.92	0.95	0.98
0.20	0.82	0.87	0.92
0.30	0.74	0.77	0.81

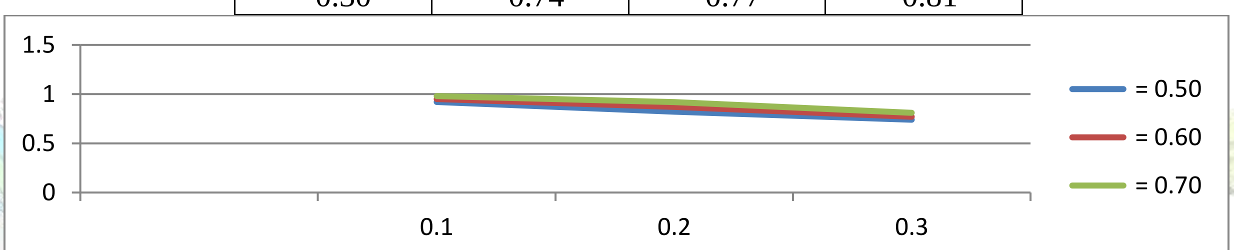

Figure 3: Availability of System

Server of busy period (B_0):-

Table 6: Server of busy period

δ \ θ	0.50	0.60	0.70
0.10	0.22	0.20	0.17
0.20	0.35	0.34	0.31
0.30	0.53	0.49	0.48

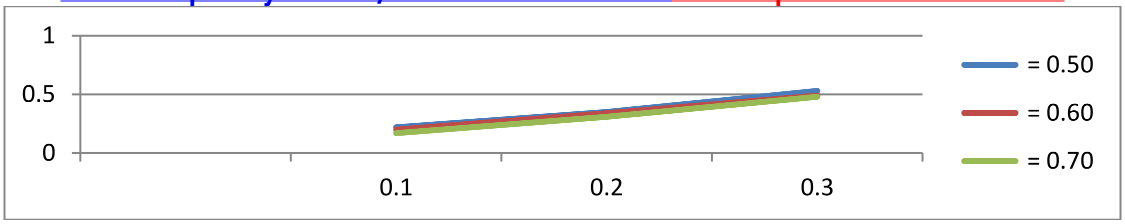

Figure 4: Server of the busy period

Expected Fractional No. of Inspection by Repairman (V_0) :-

Table 7: Expected Fractional No. of Inspection by Repairman

δ \ θ	0.50	0.60	0.70
0.10	0.21	0.22	0.23
0.20	0.28	0.30	0.31
0.30	0.42	0.46	0.51

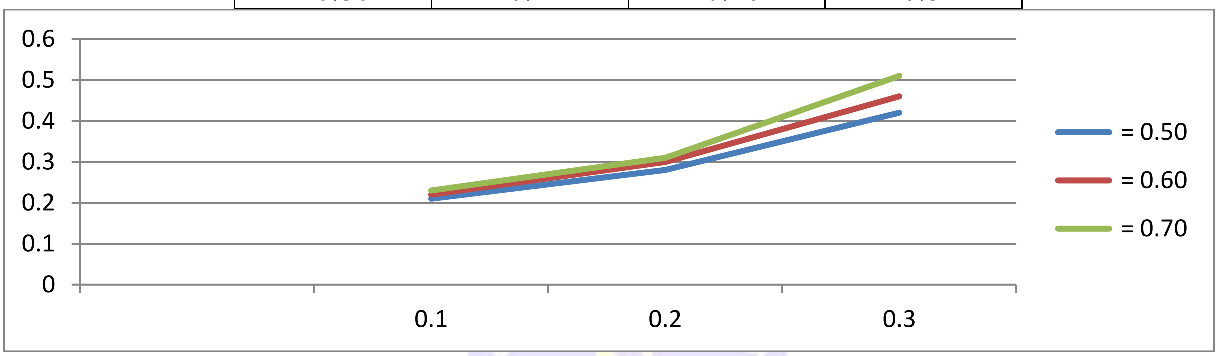

Figure 5: Expected Fractional No. of Inspection by Repairman

Conclusion: Increasing the repair rates of the unit, availability of should increase which is depicted by graphs 3 drawn, as it should be practically. From figure 4, that expending the failure rates the B_O increases. Failure rates considered as constant, for increasing the value of repair rate then the B_O decreases. It is shows that when failure rates increases V_O increase and repair rates increases thenV_O increases. The table 4 and figure 2, it is shows that when failure rates increases the MTSF decrease and repair rates increases then the MTSF decreases.

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