International Advance Journal of Engineering, Science and Management (IAJESM)
ISSN -2393-8048, January-June 2022, Submitted in May 2022, iajesm2014@gmail.com

Hyperbolic Equation with Nonlinear Programming: A Review

Bhoopendra Pratap, Research Scholar, Department of Mathematics, North Eastern Regional Institute of Technology Itanagar, Arunachal Pradesh
Prof. B. K. Singh, Department of Mathematics, North Eastern Regional Institute of Technology Itanagar, Arunachal Pradesh
Dr. Raghwendra Singh, Assistant Professor, Department of Mathematics, CSJMU, Kanpur

ABSTRACT

The mathematical theory of quasilinear hyperbolic equations places a significant emphasis on the concept of characteristic hypersurfaces and the geometry of which they are composed. It is possible for a continuous solution to have Lipschitz discontinuities in its first or higher order normal derivatives over these hypersurfaces. This is due to the fact that, in the event that characteristic hypersurfaces are present, they perform the same role as bearers of discontinuities as they do in the case of a differentiable (smooth) solution. Assuming that there is only one spatial dimension and one time, these characteristic hypersurfaces may be reduced to families of characteristic curves on the (x, t) plane. This happens when there is only one time dimension. In a derivative of the solution that is normal to the characteristics, it is possible to transport a Lipschitz discontinuity along any one of these families of characteristic curves. This is a possibility. When this occurs, the hypersurface of the solution itself transforms into a typical smooth surface. The presence of a Lipschitz discontinuity in the first derivative of the solution, which is normal to a characteristic curve, is indicated by the presence of a crease on this surface.

INTRODUCTION

The mathematical theory of quasilinear hyperbolic equations places a significant emphasis on the concept of characteristic hypersurfaces and the geometry of which they are composed. It is possible for a continuous solution to have Lipschitz discontinuities in its first or higher order normal derivatives over these hypersurfaces. This is due to the fact that, in the event that characteristic hypersurfaces are present, they perform the same role as bearers of discontinuities as they do in the case of a differentiable (smooth) solution. Assuming that there is only one spatial dimension and one time, these characteristic hypersurfaces may be reduced to families of characteristic curves on the (x, t) plane. This happens when there is only one time dimension. In a derivative of the solution that is normal to the characteristics, it is possible to transport a Lipschitz discontinuity along any one of these families of characteristic curves. This is a possibility.

The undisturbed solution that is located in front of the wave front can be considered to be the solution that is on the side of the wave front that results in propagation. On the other hand, the solution that is located on the opposite side of the wave front can be considered to be a disturbance wave that is propagating and entering a region that is occupied by the undisturbed solution. If you look at the shock front from the front, you will always see that it is moving at supersonic speed; but, if you look at it from the back, you will see that it is moving. The nonlinearity that is present in hyperbolic equations gives birth to a wide variety of wave-like behaviors in the field of applied mathematics of a wide variety of various kinds. Apart from the loss of superposibility of solutions and the consequent loss of the majority of the instruments of linear analysis, the basic concerns that occur when nonlinear hyperbolic equations are taken into consideration are the evolution and characteristics of solutions.

This is in addition to the fact that the tools of linear analysis are lost. Discontinuity in the solution is one of the inherent properties of nonlinear hyperbolic partial differential equations. This is one of the aspects that makes these equations unique.

RESEARCH OBJECTIVES

In accordance with what was discussed before, the equations that control the motion of a compressible fluid are hyperbolic. As a consequence of this, it is not always possible to determine an accurate explanation for the phenomena that are associated with fluid motion. In light of this, it is of the utmost importance to look for approximative analytical and numerical approaches whose primary objective is to accomplish this particular task. The purpose of the proposed proposal is to investigate the mathematical and physical elements of problems that are associated with the interaction and propagation of nonlinear waves in a variety of gasdynamics regimes. The research effort that is being suggested would accomplish the following::

1. Gaining a knowledge of the hyperbolic system of conservation laws and the mathematical and physical behavior of Euler's equations guiding the motion of a compressible fluid.
2. Long-term solution development from starting data studied analytically and numerically to look for non-breaking solutions.

REVIEW OF LITERATURE

In order to define the general theory of the propagation of weak discontinuities in solutions to quasilinear hyperbolic systems and to establish the time at which shock waves are formed, a number of different approaches are necessary. These approaches consist of the asymptotic method, the wavefront expansion method, the parameter expansion technique, and the wavefront analysis method. The single surface method is also included in this category. It is possible that these many ways are effective, but it is contingent upon the number of dependent and independent variables, the number and structure of the equations that link them, and the required form of the evolution rule. A significant number of authors have conducted in-depth research on the instant when shock waves are created.

In his 1978 work, Zierep offers a comprehensive summary of gasdynamics. The broad theory of shock wave propagation was conceptualized and presented by Boillat. Shifrin undertook an investigation on the formation of a shock wave in a perfect gas that was moving in a planar manner. An investigation was conducted by Ardavan-Rhad into the propagation of a plane shock wave into a medium that was not viscous, was not isentropic, and did not facilitate heat transmission.

A symmetric two-way traffic flow was utilized by Saldatov to compute the instant at which a shock wave develops. This was accomplished through the use of the Riemann technique. Several researchers, including Fusco, Germain, Fusco and Engelbrecht, Sharma et al. , Singh et al., and Nath et al., have utilized the asymptotic method in order to investigate the non-linear wave propagation in various gaseous mediums.

Using a hyperbolic system of balancing laws that is typically dissipative and quasi-linear, Ruggeri (1999) investigated the shock wave structure solutions of the system. In most cases, wave fronts converge due to the fact that they propagate in a normal direction to one another. It is possible to describe weak shocks as shocks that have a weak strength. One of the most serious problems is weak shock focusing. This issue of weak shock focussing was investigated by Wanner and colleagues.

Additionally, it is well known that those who witnessed atomic bombs observed tremendous shock waves that are referred to as blast waves. Bombardment waves are characterized by the fact that the pressure behind the shock wave is often higher than the pressure in front of it. This results in the shock wave becoming particularly powerful. Because of this, the problem is phrased in terms of similarities between the two.

In the year 1942, Guderley was the one who carried out the first investigation on converging shock waves. In order to find a solution to the problem, Guderley emphasized that the selfsimilar issue is constructed on the basis of specific physical assumptions. He also emphasized that the similarity exponent, which describes the features of a shock wave, such as its space-time trajectory near the collapse point, has to be found. The interaction of two waves colliding, the interaction of one wave crossing over another, and the interaction of one wave going into contact with a discontinuity are all examples of elementary wave interactions. Choquet-Bruhat recently presented a method that may be utilized for the purpose of addressing shockless solutions of hyperbolic systems that are dependent on a single phase function.

Ideal shock waves, also known as discontinuity surfaces that are traversed by mass fluxes, are typical examples of systems that are coupled to quasilinear hyperbolic conservation law systems. A moving surface of this kind divides the physical space into two subspaces, each of which has a solution that is continuous but jumps over the shock. The jump relations, which are often commonly referred to as the Rankine–Hugoniot conditions, are the result of the conservation laws. It is via this process that they create a connection between the field variables that were present before and after the shock and the normal velocity of the discontinuity surface. When selecting shocks that are acceptable, it is common practice to include an additional "entropy inequality" in the considerations. Consider, for example, the Hopf equation for a function that is previously unknown. $u\left(t,x\right)$ (the selection of the conservative form is a presumption that is made after the fact): $\frac{\partial u}{\partial t}+\frac{\partial}{\partial x}\left(\frac{u^2}{2}\right)=0$ and the corresponding Riemann problem $u\left(0,x\right)=\begin{cases}u^- & \text{if } x<0\\ u^+ & \text{if } x>0\end{cases}$ If the 'entropy' inequality $u^- > u^+$ maintains, a shock that is traveling with the velocity is an example of a unique discontinuous solution. $D=\left(u^- + u^+\right)/2$: $u\left(t,x\right)=\begin{cases}u^- & \text{if } x<Dt,\\ u^+ & \text{if } x>Dt\end{cases}$

Recent developments have led to the development of shock-like solutions in one dimension for a continuum in which the internal energy is dependent on both the material derivative and the density of the continuum. These answers established a connection between a stationary state and a periodic solution of the equations that control the system. The shock was found to satisfy the generalized Rankine–Hugoniot criterion, which was previously believed to be a limit with no dispersion involved. The classical laws for the conservation of mass and momentum include these requirements, which are an extra condition for the one-sided derivatives of the density that emerge from the variation of Hamilton's action. Therefore, these requirements are an additional condition. It has been demonstrated through numerical tests that this extra criterion is of great significance.

The Riemann solver that Godunov developed in the beginning is arduous and computationally inefficient when compared to the standards of today. After some time had passed, Godunov proposed an extra exact Riemann solution. Within the context of the Riemann problem, Smoller was able to solve an expanded class of hyperbolic systems that had arbitrary constant states. A comprehensive analysis of the explicit solution to the Riemann issue is presented by Toro, Schleicher, and Pike.

It is of utmost significance to find an accurate solution to the Riemann issue. For example, it is the essential building block for the well-known Glimm random choice approach, which is used to produce answers to wide starting value concerns. This method is used to generate solutions to problems. Although Godunov and Chorin have presented precise solutions to the Riemann problem, Smoller has proposed a technique that is quite different from what Godunov and Chorin have proposed.

A solution to the Riemann problem was discovered by Liu and Sun for a class of linked hyperbolic systems of conservation laws that included delta starting data. In the case of van der Waals gases, Ambika and Radha conducted research to determine whether or not an elementary wave solution to the Riemann problem exists and whether or not it is unique. Throughout the work, Bressan provided an independent introduction to the mathematical theory of hyperbolic systems of conservation laws. He concentrated particularly on the study of discontinuous solutions, which are solutions in which shock waves express themselves. In their research, Mentrelli and Ruggeri investigated the phenomenon of shock and rarefaction waves in hyperbolic models of incompressible fluids. In their study, Mentrelli and colleagues provided a comprehensive analysis of the topic of wave interaction that arises from Riemann problems in an Euler fluid.

Boillatt and Ruggeri (1979) examined the evolution of weak discontinuities for quasilinear hyperbolic systems.

In addition, the shock structure has furthermore been subjected to a considerable level of growth. Goldman and Sirovich (1969), Boillatt and Ruggeri (1998), and Kuznetsov (1979) are three individuals who have made substantial contributions to the study process on shock structure. Wavefronts that are concave in the direction of propagation exhibit a range of

behaviors, the specifics of which are determined on the strength of the wavefronts. In most cases, wave fronts converge due to the fact that they propagate in a normal direction to one another. It is possible to describe weak shocks as shocks that have a weak strength. One of the most serious problems is weak shock focusing.
During the year 1972, Wanner and colleagues carried out research on the topic of mild shock focusing. In addition, it is common knowledge that individuals who witnessed atomic bombs have reported experiencing tremendous shock waves that are referred to as blast waves. It was demonstrated by Ram (1981) that a closed form self-similar solution may be found for an MHD flow that has been disrupted by broadcast blast waves. Additionally, the shock becomes more intense during the latter stages of the collapse, and the pressure that is moving forward is deducted from the pressure that is moving behind the shock wave. Because of this, the problem is phrased in terms of similarities between the two.

A NON-LINEAR HYPERBOLIC EQUATION

The physical source of the problem under consideration is an extendable beam of length, whose ends are held a certain distance apart, hinged or clamped, and are either stretched or compressed by an axial force. The difficulty stems from the theory of vibrations of such a beam. Both translational and rotational motions are carried out by a beam's constituent parts during vibration; for further information, see Timoshenko. This phenomenon's occurrence is taken into account in this hypothesis.
An initial-boundary value problem for the non-linear hyperbolic equation is used to give a mathematical model for this problem.

$$\frac{\partial^2 u}{\partial t^2} - \lambda \frac{\partial^4 u}{\partial t^2 \partial \chi^2} + \frac{\partial^4 u}{\partial \chi^4} - [\alpha + \int_0^{\ell} [\frac{\partial u}{\partial s}(s,t)]^2 ds] \frac{\partial^2 u}{\partial \chi^2} = 0,$$ ....(e.q.)

where $u(\chi,t)$ is a real constant that is proportional to the axial force exerted on the beam when it is forced to lay along the x axis, is the deflection of point l at time t, and λ is a nonnegative constant $(y^{\lambda} = 0$ means neglecting the rotatory inertia, while $\lambda > 0$ implies taking it into consideration). The equation's nonlinearity results from taking the beam's extensibility into account. When using this model, $\lambda = 0,$ was addressed by Medelros, Ball, and Dickey in a formulation in Hilbert space. See Pohozaev, Lions, Menzala, and Rivera for similar issues.
For a Cauchy problem in a Hilbert space H, a theorem of existence and uniqueness of weak solution is established for the equation

$$(I + \lambda A)u'' + A^2 u + [\alpha + M(|A^{\frac{1}{2}} u|^2)] Au = f,$$ ....(e.q.)

with appropriate constraints on operator A and the supplied functions M and f. There are three sections to this study. The theorem is proven and the existence of a weak solution is established in Part I. Its distinctiveness is shown in Part. Lastly, a request is made in Part when H is $L^2(\Omega)$, Ω a regular border bounded open set in R^n, and A is the Laplace operator $-\Delta$.

Existence of weak solution

Let H be an inner product of real Hilbert spaces. $(\ ,\ )$ and norm $|\ |$.
Let A be a linear operator in H, and let V be the dense domain of A in H. Using the graph norm of A as a reference $||\ \ ||$, i.e.

$$|| v ||^2 = |v|^2 + |Av|^2 , \text{ for } v \text{ in } V,$$

V is a continuous injection of a real Hilbert space into H. This injection is assumed to be compact.

Assume Positive and self-adjoint, that is, there is a consistent $k > 0$ such that

$(Av,v) \geq k|v|^2$, for v in V. ....(e.q.)

Denote the pairing between V' and V by letting V' be the dual of V. Once H and H' have been identified, V c H c V'. Given that injections are dense and continuous, it is known that (h, v) for h in H and v in V.

Define A^2: $V \rightarrow V'$ by

$< A^2u, v > = (Au,Av)$, for u, v in V.

It follows that A^2 is a linear operator with bounds from V to V'..

Denote the bilinear form in this way: a(u,v). $D(A^{\frac{1}{2}})$ associated to A, i.e.,

$a(u,v) = (A^{\frac{1}{2}}u, A^{\frac{1}{2}}v)$, for u, v in $D(A^{\frac{1}{2}})$

a(u) means a(u,u).

Given $\lambda \geq 0$, consider in $W = D((\lambda A)^{\frac{1}{2}})$ the graph norm of $(\lambda A)^{\frac{1}{2}}$, denoted $|\ \ |_\lambda$, i.e.,

$|w|_\lambda^2 = |w|^2 + \lambda|A^{\frac{1}{2}}w|^2$, for w in W

Note that W H, if $\lambda = 0$, and $W = D(A^{\frac{1}{2}})$, if $\lambda > 0$; V is hence dense in W. Let be a true number, M is a true C^1 function, with $M'(s) \geq 0$, for $s \geq 0$.

Assume that mo and m I are positive constants such that $M(s) \geq m_o + m_1 s$, for s > 0.

Observe that if M is the identical function, the substitution of $\alpha + s$ by $(\alpha - m_o) + (m_o + s)$, with arbitrary $m_o > 0$, guarantees that M satisfies the aforementioned criteria.

Now, the theorem may be expressed..

Theorem. Given f in $L^2(0,T;H)$, u_o in V, u_1 in W, there is a unique function $u = u(t)$, $0 \leq t < T$, such that:

a) $u \in L^\infty(0,T;V)$

b) $u' \in L^\infty(0,T;W)$

c) U is a feeble remedy for

$(I + \lambda A)u'' + A^2u + [\alpha + M(|A^{\frac{1}{2}}u|^2)]\ Au = f$, ...(e.q.)

In other words, u fulfills in D' (0,T) for any v in V.

$$\frac{d}{dt}[(u'(t),v) + \lambda a(u'(t),v)] + (Au(t),Av) +$$

$$+ [\alpha + M(a(u(t)))]\ a(u(t),v) = (f(t),v) ,$$ ....(e.q.)

d) The ensuing preconditions are met:

$u(0) = u_0 \ , \quad u'(0) = u_1$ ....(e.q.)

Prior to demonstrating the theorem, a few important observations.

IDEAL AND NON-IDEAL GAS

There are no interactions between the particles that make up a hypothetical gas that is referred to as an ideal gas. This gas is composed of a number of particles that move by chance. Several real gases exhibit characteristics that are qualitatively similar to those of an ideal gas under a variety of pressure and temperature conditions. In these cases, the molecules of the gas (or atoms, in the case of monatomic gases) behave as if they were the ideal particles. When intermolecular interactions and molecular size become substantial, the ideal gas model often fails to hold up when the temperature is lower or the pressure is higher. In addition, it is not compatible with the majority of heavy gases, which includes a great number of refrigerants, as well as gases that have substantial intermolecular interactions, such as water vapor. It is common for the volume of a real gas to be much more than the volume of an ideal gas when the pressure is high. In many cases, the pressure of a real gas is significantly lower than the pressure of an ideal gas when the temperature is low. The transformation of real gases into other phases, such as liquids or solids, occurs when the temperature and pressure are exactly right. However, the ideal gas model does not adequately represent phase transitions and does not allow for their occurrence. These need more complex equations of state to be modeled than the previous ones. If, on the other hand, the temperature is too high and the density is abnormally low, then the premise that the gas is ideal becomes called into question. In that case, the only choice is to relax the assumptions regarding the ideal gas.

There are two gas laws that have been discovered by testing, and they are expansions of the ideal gas law. These laws are Charles' law and Boyle's law. The equation of state for an ideal gas is written as PV = nRT, where n represents the molecular count of the gas, R represents the gas constant, T represents the gas's absolute temperature, P represents the gas's pressure, and V represents the gas's volume.

The real gases equation is $\lim_{P\to\infty} PV/RT = 1$ Moreover, the compressibility factor Z is described as

$Z(P,T) = PV/RT$ . ...(e.q.)

MAGNETOGASDYNAMICS

Magnetogasdynamics is yet another key example that illustrates the theory of the hyperbolic system. Because the governing system of magnetogasdynamics is so complex and highly non-linear, it is essential to conduct an analysis of a variety of simpler models in which the magnetic field and the velocity field are always orthogonal to one another. Magnetic fields are present throughout the universe and play a significant role in a variety of astrophysical situations. It is highly probable that these magnetic fields have an effect on all astrophysical plasmas. The usage of shock waves in magnetic fields has a wide range of applications in the industrial sector. Magnetic fields are involved in a wide variety of exciting astrophysical and aerodynamical challenges. Activities that release a considerable amount of energy in a short amount of time might potentially result in the production of cylindrical shock waves. Magnetic fields are utilized in a variety of scientific domains, including astrophysics, geophysics, plasma physics, and others. The universe is filled with magnetic fields. The magnetic field has an effect on a wide variety of astrophysical phenomena, including as gamma-ray bursts, magnetized stellar winds, synchrotron radiation from supernova remnants, and the formation of the interstellar medium, which plays a particularly important role in the formation of stars, galaxies, and galaxy clusters. Through their articles, Hartmann and Balick investigated the process by which star clusters are formed. A number of additional notable research on the propagation of shock waves in a magnetic field are also included in this compilation.

WEAK SHOCK WAVES IN NON-IDEAL GAS FLOW WITH RADIATION AND WEAKLY NON-LINEAR RESONANT WAVES

For the purpose of gaining practical insights into the complex physical processes that are the subject of investigation, it is essential to have a solid understanding of the asymptotic solution of the system of quasilinear hyperbolic PDEs. Through the application of the theory of progressive wave analysis, a numerous number of researchers have investigated the propagation of weakly non-linear waves in a variety of gaseous media. In the study of weak shock wave propagation, which is represented as the hyperbolic system of partial differential equations (PDEs), asymptotic analysis is a method that is widely used. Through the use of the ray approach, were able to get the solution for the hyperbolic system that was weakly nonlinear and utilized high frequency waves. Several researchers, have used the asymptotic method in order to investigate the non-linear wave propagation in various gaseous media. The vast majority of natural physical processes may be comprehended via the use of mathematical models that are founded on the hyperbolic system of partial differential equations. It who used the perturbation strategy in order to get the shockless solution of a hyperbolic system of partial differential equations that was based on a single phase function.

The propagation of discontinuous waves under the influence of radiation has been the subject of research conducted by a great number of specialists in great detail. In the past, a number of investigations have been conducted on the asymptotic properties of weak shock waves in various gas dynamic regimes. In these regimes, the governing equation is a set of quasilinear hyperbolic partial differential equations (PDEs).

NON-LINEAR GEOMETRICAL ACOUSTICS SOLUTION AND PROGRESSIVE WAVE SOLUTION WITH RIEMANN PROBLEM

It is possible to provide a mathematical explanation for a wide variety of natural occurrences, including chemical and nuclear explosions, bomb blasts, collisions between two or more galaxies, supersonic flow, and many others. This can be accomplished by employing a model of quasilinear hyperbolic partial differential equations. As a result of its numerous applications in nuclear physics, plasma physics, astrophysical sciences, and interstellar gas masses, the study of shock waves, acceleration waves, weakly non-linear waves, shock wave interaction, and other related topics has been the focus of research in non-linear science and engineering for a significant amount of time.

A significant amount of interest from academics has been shown in the asymptotic method for solving weakly non-linear hyperbolic waves throughout the course of the last several decades. By taking into consideration a shockless solution to a system of hyperbolic equations that is dependent on just a single phase function, made a significant contribution to the study of non-linear waves with tiny amplitudes. was the first person to investigate the single phase progressive wave solution for the weekly non-linear waves. Several authors, such as, have used the progressive wave approach in order to investigate the wave propagation problem in various gasdynamicregimes while conducting their research. Significant additions to this reference have been made by a number of authors, including, among others. The uniformly valid asymptotic theory of resonantly interacting high frequency waves for a non-linear hyperbolic system of equations was obtained by Hunter and colleagues in the year 1986.

FINDINGS AND CONVERSATION

Through the use of the multiple time scale technique, the system of hyperbolic partial differential equations that describes the one-dimensional unsteady, compressible planar, cylindrically symmetric, and spherically symmetric flow of a van der Waals gas is solved asymptotically for high frequencies with small amplitudes. Through the lens of weakly non-linear geometrical acoustics theory, these wave interaction conditions that occur resonantly are put under the microscope for investigation. A system of quadratic non-linearity inviscid Berger's equations is made up of the transport equations that we have inferred here. These equations are related to one another by a linear integral operator kernel that is already known.

WEAK SHOCK WAVE EVOLUTION IN NON-IDEAL MAGNETOGASDYNAMICS

Hyperbolic systems of partial differential equations are used as models to provide mathematical explanations for a wide variety of natural events that may be seen in the natural world. are some useful references to consult. One of the most important aspects of non-linear systems is the study of discontinuity waves, which include shock waves, acceleration waves, and weekly non-linear waves. This is because of the applications that discontinuity waves have in gas dynamics. Since shock waves, or discontinuities, are a common occurrence in a variety of astrophysical scenarios, including as supernova explosions, star winds, photo ionized gas, collisions between high-velocity clumps of interstellar gas, collisions between two or more galaxies, etc., and since the gas is not perfect, Numerous conducting fluids and plasma fluxes seen in both industrial and natural settings are covered by magneto hydrodynamics. In the past, a number of different approaches have been used in order to investigate the asymptotic properties of weakly non-linear waves and the manner in which waves propagate over a range of material mediums while being controlled by a quasilinear hyperbolic system of equations examined a shockless solution of a system of hyperbolic partial differential equations that depends on a single phase function. This allowed Choquet-Bruhat to concentrate a considerable amount of attention on non-linear progressive waves with a very small amplitude. were among the writers who used the perturbation technique in order to investigate the nonlinear wave propagation in a variety of material media. An approach known as the Ray method was first presented with the purpose of finding a wave solution with a high frequency and a moderate amplitude for a set of quasilinear hyperbolic partial differential equations. Under circumstances in which the temperature of the gas is excessively high or its density is excessively low, a simplified van der Waals model may be used as an alternative to the ideal gas.

RESULTS AND DISCUSSION

The quasilinear hyperbolic system of equations for an ideal polytrophic dusty gas may be solved analytically, which results in the generation of shock waves, rarefaction waves, and contact discontinuities. It is important to note that the findings obtained are identical to the results that were obtained in the past for an ideal gas and non-magnetic situation when there are no dust particles present, which is equivalent to 0. The figures illustrate the density and velocity profiles for compressive waves (one-shock and three-shock) and rarefaction waves (one-shock and three-shock) in relation to ε (psi), where δ was previously defined as logarithm, logarithm of the square of the pressure. Utilizing MATLAB to calculate the dust particle mass fraction values (p k) for a variety of dust particles in the gas For the sake of the computations, the constant values that appear are presumed.

$$\gamma = 1.4, \omega = 0.8, Z_1 = 0.03, \ Z_2 = 0.04, \ k_p = 0.0, 0.2, 0.4, 0.6. \quad \text{.................................(e.q.)}$$

CONCLUSION

Analytical solutions to the Riemann problem for the quasilinear hyperbolic system of equations of an ideal polytropic dusty gas, also known as 1-shock waves (1-simple waves) and 3-shock waves (3-simple waves), are shown and detailed in this chapter. There are no restrictions placed on the size of the initial states. A conclusion has been reached that the Riemann problem in dusty gases must have a singular solution, and that this solution must exist under circumstances that are both necessary and sufficient. In addition, the circumstances that provide information on the existence of shock waves or simple waves for curves belonging to the 1- and 3-family are investigated. When dust particles are present in an ideal polytropic gas, the expression becomes more difficult than it would be in the comparable ideal scenario; nonetheless, the discoveries that are similar to the circumstance remain the same. Furthermore, it is shown that the consequences are the same when considering the case of a perfect polytropic environment that is free of dust particles.

The small amplitude high frequency asymptotic solution to the system of hyperbolic partial differential equations describing one-dimensional unsteady, compressible planar and non-planar flow in a van der Waals gas is derived in the current study utilizing the technique of multiple time scales. A description is given of the conditions that are necessary for resonant wave interaction to take place. There are transport equations for the wave amplitude along its rays that are generated from the system of hyperbolic equations. This system is made up of inviscid Berger's equation with quadratic non-linearity, which is coupled by a linear integral operator with a known kernel

BIBLIOGRAPHY

[1] Boillatt, G., Ruggeri, T., "On the evolution law of weak discontinuities for Hyperbolic quasi-linear hyperbolic systems", Wave Motion, 1, 149-152, 1979.
[2] Boillatt, G., Ruggeri, T., "On the shock structure problem for hyperbolic system of balance laws and convex entropy", Continumm Mech. Thermo., 10, 285-292, 1998.
[3] Bowen, R. M. and Chen, P. J., "Shock waves in ideal fluid mixtures with several temperatures", Arch. Rational Mech. Anal., 53, 277-294, 1974.
[4] Branover, H., "Magnetohydrodynamics Flow in Ducts" Wiley, New York, 1978.
[5] Butler, D., "Converging spherical and cylindrical shocks", Armament Res. Estab. Rep., 54/54. 1954.
[6] Fusco, D., "Some comments on wave motions described by non homogenous quasilinear first order hyperbolic system", Meccanica, 17, 128-137, 1982.
[7] Fusco, D., Engelbrecht, J., "The asymptotic analysis of non linear waves in rate dependent media", Nuovo Ciment B, 80, 49-61, 1984.
[8] Germain, P., "Progressive waves", in: 14th L. Prandtl Memorial Lecture, Jahrbuch, der DGLR, pp. 11-30, 1971.
[9] Giacomazzo, B., Rezzolla, L., "The exact solution of the Riemann problem in relativistic magnetohydrodynamics", J. fluid Mech. 562, 223-259, 2000.
[10] Glimm, J., "Solution in the large of non-linear hyperbolic systems of equations", Comm. Pure Appl. Math, 18, 697-715, 1965.
[11] Hunter, J. K., Majda, A., Rosales, R., "Resonant interacting weakly non-linear hyperbolic waves II. Several space variables", Studies in Applied Mathematics, 75, 187-226, 1986.
[12] Hunter, J.K., "Asymptotic equations for non-linear hyperbolic waves in: M. Freidlin, et al. (Eds), surveys in applied mathematics". vol. 2 Plenum Press, New York, pp. 167-276, 1995.
[13] Jeffrey, A., "Quasilinear hyperbolic systems and waves", Pitman, London, 1976.
[14] Lax, P. D., "hyperbolic system of conservation laws II", Commun. Pure Appl. Math., 18, 697-715, 1963.
[15] Majda, A., Rosales, R., "Rosales, Resonantly interacting weakly non-linear hyperbolic waves", Studies in Applied Mathematics, 71, 149-179, 1984.
[16] Smoller, J., "On the solution of the Riemann problem with general step data for an extended class of hyperbolic systems", Michigan Math. J., 16, 201-210, 1969.