



## A Continuum Approach of Mechanics in Form of Nonlinear Equations

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### ABSTRACT

The undisturbed solution that is located in front of the wave front can be considered to be the solution that is on the side of the wave front that results in propagation. On the other hand, the solution that is located on the opposite side of the wave front can be considered to be a disturbance wave that is propagating and entering a region that is occupied by the undisturbed solution. If you look at the shock front from the front, you will always see that it is moving at supersonic speed; but, if you look at it from the back, you will see that it is moving. The nonlinearity that is present in hyperbolic equations gives birth to a wide variety of wave-like behaviors in the field of applied mathematics of a wide variety of various kinds. Apart from the loss of superposibility of solutions and the consequent loss of the majority of the instruments of linear analysis, the basic concerns that occur when nonlinear hyperbolic equations are taken into consideration are the evolution and characteristics of solutions.

### INTRODUCTION

In continuum mechanics, the rules of conservation of mass, momentum, and energy serve as a common basis. After that, the constitutive laws of each media are used to characterize the medium in question. Field equations, also known as partial differential equations, are frequently nonlinear and no homogeneous. These equations are simplified to field equations under fairly plausible assumptions. The constitutive equations and conservation rules for the field variables are simplified also. In linear theory, the main characteristics of motion and the methods that are employed to address nonlinear problems are recognized to the same extent as they are in linear theory.

Since 1930, when the study of the compressibility factor in air flow became important as a result of higher speeds in aviation, the area of gas dynamics has existed as a separate field within the science of fluid dynamics. During and after World War II, there was a significant increase in the development of gas dynamics due to the widespread application of gas dynamics in technology. This includes the development of jet aviation, rocketry, jet and rocket engines, supersonic aircraft and missiles, and the creation of nuclear bombs, the explosion of which produces powerful blast and shock waves.

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### REVIEW OF LITERATURE

#### REVIEW OF SHOCK WAVES IN GASDYNAMICS

Shock waves are a type of mechanical wave that are formed when matter is squeezed at a rapid rate. These waves have restricted vibrational amplitudes. There are four main ways in which shock waves are distinct from acoustic waves. Acoustic waves are waves that have amplitudes that are so minute that they are almost microscopic.:



- The formation of a steep wave front with an abrupt change in all thermodynamic quantities;
- A pressure-dependent, supersonic propagation velocity;
- A strong decrease in propagation velocity for nonplanar shock waves with increasing distance from the center of origin;
- Nonlinear superposition (reflection and interaction) properties..

First and foremost, shock waves are utilized in the field of aerospace engineering, namely for the purpose of supersonic flight. During the process of calculating the velocity of a bullet using a ballistic pendulum in 1746, the mathematician Robins made the discovery that there is an increase in aerodynamic drag when the velocity approaches the speed of sound. This discovery marked the beginning of this particular discipline of physics.

However, a significant number of academics were remained perplexed by the phenomenon of shock waves during the 19th century. In 1759, Euler studied the amplitude of a sound wave, sometimes known as the "size of disturbance" although he did not use the word shock waves anywhere in his discussion. Despite this, he was absolutely wrong in his assumption that the velocity would drop as the amplitude grew. The Euler equation for one-dimensional unsteady fluid flow was solved by Poisson in 1808, making him the first scientist to discover explicit solutions to the problem.

Poisson made an important contribution to the area of nonlinear wave theory in 1823 when he established the isentropic gas law for the sound wave with infinitesimal amplitude. This accomplishment was a big step forward in the field.

By virtue of their restricted amplitude, powerful explosions are examples of gas compressions that travel at a speed that is greater than the speed of sound. In recent times, the creation of shock waves has garnered a significant amount of interest in the academic literature. Both the shock generation time and the shock creation distance are critical characteristics that determine the relative relevance of convective nonlinear steepening and dissipative flattening. These parameters also set constraints on the use of various approximation theoretical techniques.

When Chadha and Jena (2015) discussed the relatively undistorted method of propagating weak disturbances in a non-ideal gas that contains dust particles, they were presenting their findings. Singh and colleagues conducted research in 2016 to investigate the problem of the propagation of moderate shock waves in an electrically conducting, inviscid fluid while the fluid was subjected to the impact of a magnetic field. The strength of a shock wave and the first-order discontinuity that it generates are both governed by a set of two connected nonlinear transport equations. These equations allow for a solution that is in accordance with the classical decay laws for a moderate shock.

The work (1) that was done by Tellegen in 1933 was the first step in the investigation of nonlinear phenomena that occurred during the propagation of electromagnetic waves in a gas that was only slightly ionized. It was brought to everyone's attention that the loud signal from the Luxembourg station was interfering with broadcast signals in the ionosphere through the process of cross-modulation. The term "cross-modulation" is now often used to describe the phenomenon known as the Luxembourg effect. This phenomenon arises as a consequence of the nonlinear interaction between two electromagnetic waves in the ionosphere when the disturbing wave is varied around its amplitude.

For the purpose of providing an explanation for Tellegen's results, Bailey and Martyn (2) took into consideration the impact of an electromagnetic wave that was passing by on the collision frequency. This, in turn, has an effect on the propagation of another wave in the disturbed medium. According to their hypothesis, there will be a cross-modulation that can be detected, and the transmission signals and the ionosphere's physical characteristics will have numerical values that are already known. Both the reduced conceptions of the effective collision frequency of the free electrons and the geometrical optics approximation were utilized by them while conducting their research.



As opposed to approaching the problems using the mean free path or relaxation time technique as in (2), the majority of researchers prefer to use statistical mechanics to solve Boltzmann's kinetic equation for the electron distribution function in an ionized gas in the presence of disruptive electromagnetic waves. This is because statistical mechanics allows them to solve the equation more efficiently. By applying their knowledge of the electron distribution function, they are able to ascertain the nonlinear conductivity tensor of the gas that has been disrupted. In the presence of an electric and magnetic field, Chapman and Cowling (3) were able to obtain the Boltzmann kinetic equation of motion for an ionized gas. Additionally, they were able to estimate the collision term for both elastic and inelastic collisions.

The electron distribution function for a plasma in the presence of electromagnetic waves and a magnetostatic biasing field was later found by Fain and Sodha. It was supposed that elastic electron-molecule collisions predominated because the plasma was lightly ionized. In addition, Sodha computed the electric convection current density in the nonlinear plasma that was disrupted by the electromagnetic waves that were traveling past. Ginzburg (12) talked about how a magnetostatic field, non-uniformity in the plasma, and changes in collision frequency may all cause a plasma to become nonlinear. He and Gurevick provided a thorough examination of the cross-modulation of two electromagnetic waves and nonlinear processes in a plasma situated in an electromagnetic field.

The time-invariant change in a plasma's complex conductivity caused by the passing electromagnetic waves' heating influence on the electrons was determined by Sodha and Palumbo. They then went on to look at the nonlinear interaction of many electromagnetic waves as well as the nonlinear propagation of an amplitude-modulated electromagnetic wave in a prism. It was anticipated that the waves would propagate in the same direction and without the assistance of an outside magnetostatic field. Papa (17) determined the nonlinear complex conductivity tensor of a magneto-active plasma in relation to the radiowave's polarization in a recent article. Next, he determined the radio frequency propagation coefficients of reflection and transmission in an inhomogeneous, magnetoactive, nonlinear plasma. The propagation direction was assumed to be parallel to the external magnetostatic biasing field in that particular location. These writers only included TEM waves in their list of electromagnetic waves. Because electromagnetic waves often propagate at an angle to the direction of the earth's magnetic field, this constraint restricts the utility of their ideas when applied to the propagation of radio waves in the ionosphere, where the earth's magnetic field cannot be ignored.

From the intrinsically nonlinear Boltzmann kinetic equation, it will be discovered in this work that there are two distinct nonlinear effects: the heating effect of the waves as they pass by the electrons and the spatial dispersion effect caused by the longitudinal components of the waves' electric field. Wave form distortion in passing waves is caused by the heating impact on electrons, whereas harmonic or mixed frequency waves are generated via spatial dispersion. Prior research has limited the waves to TEM waves with zero longitudinal components of the electric field along the path of propagation. This eliminates the influence of spatial dispersion and prevents the creation of waves at mixed or harmonic frequencies. Macpherson (1971) used a molecular-dynamic technique to study shock wave production in thick Argon. The creation of a shock wave in a perfect gas flowing in a plane was investigated by Shifrin (1970).

**RADIATING GAS**

In the field of radiative gasdynamics, the basic equations that are responsible for controlling the flow are made up of a sequence of coupled integro-differential equations that are rather difficult. As a consequence of these problems, there has been an intensified search for an approximation formulation of the radiative transfer equation, which ultimately leads to a system of nonlinear differential equations. In the field of radiative gasdynamics, the basic equations that are responsible for controlling the flow are made up of a sequence of coupled



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**MAGNETOGASDYNAMICS**

There are many distinct approaches to exploring and solving nonlinear partial differential equations (PDEs), and these approaches have been recorded in the published literature. The method known as dimensional analysis is one that is utilized rather frequently within the field. Lie's transformation theory of differential equations provides the foundation for an alternate method of approaching the problem. Despite the fact that the dimensional analysis technique has shown to be highly successful, Blummann and Cole discuss some of the limitations associated with it. Sophus Lie's Lie group analysis offers a powerful and straightforward approach to identifying the one-of-a-kind category of solutions for nonlinear partial differential equation (PDE) systems. For literature on differential equations Lie group invariance approaches, one can look at the available literature. The investigation of all classes of similarity solutions for one-dimensional, time-dependent shock hydrodynamics was carried out by Logan and Perez with the assistance of group-theoretic methods. This investigation focused on how the chemical reaction occurs behind the shock front. Coggeshall and Axford obtained the similarity solutions by utilizing the group invariance characteristics of a system of radiation hydrodynamic partial differential equations (PDEs).

**WEAK SHOCK WAVES IN NON-IDEAL GAS FLOW**

For the purpose of gaining practical insights into the complex physical processes that are the subject of investigation, it is essential to have a solid understanding of the asymptotic solution of the system of quasilinear hyperbolic PDEs. Through the application of the theory of progressive wave analysis, a numerous number of researchers have investigated the propagation of weakly non-linear waves in a variety of gaseous media. In the study of weak shock wave propagation, which is represented as the hyperbolic system of partial differential equations (PDEs), asymptotic analysis is a method that is widely used. Through the use of the ray approach, were able to get the solution for the hyperbolic system that was weakly nonlinear and utilized high frequency waves. Several researchers, have used the asymptotic method in order to investigate the non-linear wave propagation in various gaseous media. The vast majority of natural physical processes may be comprehended via the use of mathematical models that are founded on the hyperbolic system of partial differential equations. It who used the perturbation strategy in order to get the shockless solution of a hyperbolic system of partial differential equations that was based on a single phase function.

The propagation of discontinuous waves under the influence of radiation has been the subject of research conducted by a great number of specialists in great detail. In the past, a number of investigations have been conducted on the asymptotic properties of weak shock waves in various gas dynamic regimes. In these regimes, the governing equation is a set of quasilinear hyperbolic partial differential equations (PDEs). The research conducted by Nath and colleagues looked into the analysis of shock-related phenomena in non-ideal gas when radioactive heat transfer or magnetic field was present. investigated the propagation and decay behavior of moderate shock waves in an inviscid fluid with the extra impact of a magnetic field.

**DUSTY GAS FLOW IN ONE-DIMENSIONAL: THE INTERACTION OF WAVES**

According to a quasilinear hyperbolic system of partial differential equations (PDEs), waves move across a medium. There are several discontinuities that are often encountered. These discontinuities are known as shock waves, weak waves, and acceleration waves. Due to the fact that these waves are used in a wide range of domains, such as nuclear physics, plasma physics, geophysics, astrophysical sciences, and interstellar gas masses, the investigation of these waves has been of great significance to the fields of engineering science and nonlinear science. In this chapter, we investigate the evolutionary behavior of shock waves by using the asymptotic analysis approach, which has been applied extensively over the course of many



decades by a number of researchers, including,. Furthermore, in order to get a qualitative description of the interaction of non-linear waves, it is possible to make use of the interaction coefficients that are included in the transport equation.

Over the course of the last few decades, a number of studies have been conducted to investigate the asymptotic characteristics of shock waves in a variety of gasdynamic regimes. These regimes are characterized by a set of quasilinear hyperbolic partial differential equations that represent the governing equation. It is possible to manage the interaction of non-linear high frequency small amplitude waves in a systematic manner with the assistance of the "Weakly non-linear geometrical acoustics theory." Despite the fact that the phenomenon of wave propagation with the extra impact of nonlinearity has been explored in the past, the closed form correct analytical solution of the equations regulating wave motion has never been discovered. Numbers of approximate analytical responses are the only topics that are discussed in the vast majority of the published works. In this particular setting, it is important to highlight the contribution made by a number of authors.

**NON-LINEAR RESONANT WAVES WITH WEAKNESS**

The multiple time scale technique will be used in this section to determine the high frequency small amplitude asymptotic solution to the system of This will be done in the event that the attenuation time scale ( $\tau_{at}$ ) is greater than the characteristic time scale ( $\tau_{ch}$ ).

$\xi = \tau_{ch}/\tau_{at} \ll 1$ . Let  $l^{(i)}$  and  $r^{(i)}$  ( $i = 1, 2, 3$ )

Both the left and right eigenvectors of the matrix are shown, and they correspond to the eigenvalues  $\lambda_1 = c_0$ ,  $\lambda_2 = 0$ , and  $\lambda_3 = -c_0$ ,

respectively. The vectors that represent the eigenvalues  $l^{(i)}$  and  $r^{(i)}$  ( $i = 1, 2, 3$ ) fulfill the

requirements for normalization  $l^{(i)}r^{(j)} = \delta_{ij}$  ( $1 \leq i \leq 3, 1 \leq j \leq 3$ ), where  $\delta_{ij}$  is It was Delta Kronecker. In light of the aforementioned assumptions, the left and right eigenvectors may now be expressed as

$l^{(1)} = (0, \frac{\rho_0}{2c_0}, \frac{1}{2c_0^2})$ ,  $r^{(1)} = (1, \frac{c_0}{\rho_0}, c_0^2)$ ,  
 $l^{(2)} = (-c_0^2, 0, 1)$ ,  $r^{(2)} = (\frac{-1}{c_0}, 0, 0)$ ,  
 $l^{(3)} = (0, \frac{-\rho_0}{2c_0}, \frac{1}{2c_0^2})$ ,  $r^{(3)} = (1, \frac{-c_0}{\rho_0}, c_0^2)$

.....(e. q.)

**NON-LINEAR SOLUTION FOR GEOMETRIC ACOUSTICS**

An approximate asymptotic solution of type exists for the system of equations or, and it is able to satisfy the tiny amplitude oscillating initial data that is provided by

$U(x, 0) = U_0 + \xi U_1^0(x, x/\xi) + O(\xi^2)$ , .....(e. q.)

**NON-LINEAR GEOMETRICAL ACOUSTICS SOLUTION**

A significant amount of interest from academics has been shown in the asymptotic method for solving weakly non-linear hyperbolic waves throughout the course of the last several decades. By taking into consideration a shockless solution to a system of hyperbolic equations that is dependent on just a single phase function, made a significant contribution to the study of non-linear waves with tiny amplitudes. was the first person to investigate the single phase progressive wave solution for the weekly non-linear waves. Several authors, such as, have used the progressive wave approach in order to investigate the wave propagation problem in various gasdynamic regimes while conducting their research. Significant additions to this reference have been made by a number of authors, including, among others. The uniformly valid asymptotic theory of resonantly interacting high frequency waves for a non-linear hyperbolic system of equations was obtained by Hunter and colleagues in the year 1986.

You should consult. Through the use of the progressive wave approach, conducted an investigation of the basic feature of weakly non-linear waves experienced in non-ideal gas. In



addition, used an approach that was quite similar to this one in order to investigate the property of weakly non-linear waves in van der Waal gas. Researchers investigated the wave interaction in a non-equilibrium gas flow by using an approach that included many time scales. Using the same method, were able to accomplish the asymptotic solution of the system of hyperbolic equations in a variety of various media. Additionally, the method of weakly non-linear geometrical optics is used in order to investigate the geometry of fast magneto sonic rays as well as the creation of high-frequency fast magneto sonic waves that propagate into an ax symmetric equilibrium plasma.

FINDINGS AND CONVERSATION

Through the use of the multiple time scale technique, the system of hyperbolic partial differential equations that describes the one-dimensional unsteady, compressible planar, cylindrically symmetric, and spherically symmetric flow of a van der Waals gas is solved asymptotically for high frequencies with small amplitudes. Through the lens of weakly non-linear geometrical acoustics theory, wave-wave interaction conditions that occur resonantly are put under the microscope for investigation. A system of quadratic non-linearity inviscid Berger's equations is made up of the transport equations that we have inferred here. These equations are related to one another by a linear integral operator kernel that is already known. The neighboring coefficients that appear in the transport equations provide us with qualitative information about the interaction process that is taking place there, in addition to providing a measurement of the degree to which the various models are coupled to one another. the transport equation that we have discovered here for the planar is identical to the transport equation that was previously calculated by other writers. This is something that should be taken into consideration. The influence of the van der Waals parameter is included into the solution by means of the parameter A0. The make it abundantly evident that a rise in the density of high-frequency waves with small amplitudes occurs when the value of b increases while the value of a remains constant. Additionally, growing values of b have the same effect on the pressure and velocity of these waves as they do on the waves themselves.

SLIGHT WAVES

In a system of hyperbolic partial differential equations in one-dimensional space, a centred rarefaction wave is a simple wave. This kind of wave occurs when the dependent variables remain constant throughout the characteristics, and when one set of characteristics is composed of straight lines. The two stable requirements that pertain to a rarefaction wave are

as follows:  $U_1^*$  and  $U_2^*$  satisfy the following characteristics and are connected by a transition that is smooth in a characteristic field k that is really non-linear. (i) The Riemann invariants do not change during the wave; (ii) The wave's left and right characteristics, that is,

the wave's left and right characteristics, diverge.  $\lambda_k(U_1^*) < \lambda_k(U_2^*)$ ,  $k=1,3$ . Next, we will calculate the fundamental wave curves. One-simple waves are the only ones that are investigated in this article; the particulars of three simple waves are equivalent.

Given that a one-simple wave has a one-Riemann invariant that is constant, we are able to get

$S_2 = S_1,$   
.....(e. q.)

and

$u_2 + \frac{2C_2}{(\Gamma-1)}(1-Z_2) = u_1 + \frac{2C_1}{(\Gamma-1)}(1-Z_1) .$   
.....(e. q.)

We have the equation of state as the gas is polytropic ideal with dust particles.

$p = ke^{S/c} (\rho/(1-\theta\rho))^\Gamma .$   
.....(e. q.)

Thus, we get from





$$\frac{p_2}{p_1} = \left(\frac{C_2}{C_1}\right)^{\frac{2\Gamma}{\Gamma-1}} \left(\frac{1-Z_2}{1-Z_1}\right)^{\frac{2\Gamma}{\Gamma-1}} = \left(\frac{\rho_2}{\rho_1}\right)^{\Gamma} \left(\frac{1-Z_1}{1-Z_2}\right)^{\Gamma} \dots\dots\dots(e. q.)$$

Moreover, we get from

$$\frac{u_2 - u_1}{C_1} = \frac{2}{\Gamma - 1} \left[ (1 - Z_1) - \frac{C_2}{C_1} (1 - Z_2) \right] \dots\dots\dots(e. q.)$$

But  $\lambda_1 = u - C$  must rise in a wave of one rarefaction so  $\lambda_1^{(2)} \geq \lambda_1^{(1)}$  gives  $u_2 - u_1 \geq C_2 - C_1$ .  
Therefore, using, we get

$$\frac{C_2 - C_1}{C_1} \leq \frac{2}{\Gamma - 1} \left[ (1 - Z_1) - \frac{C_2}{C_1} (1 - Z_2) \right], \dots\dots\dots(e. q.)$$



**CONCLUSION**

In this study, the key properties of weakly non-linear waves that propagate in a compressible, inviscid, non-ideal radiating gas and dusty gas flow are investigated by the use of the progressive wave analysis methodology. The disturbance propagation in the high frequency domain is characterized by an evolution equation that we present. This equation also defines the criteria for the development of a shock wave within a restricted period of time. At the front, a shock wave that is sufficiently mild is collected, and its motion is analyzed as a sawtooth wave, also known as a half-N wavelength. We provide a description of the sawtooth wave length and velocity for planar and cylindrically symmetric flows in non-ideal radiating and dusty gas, where we also provide a visual representation of these values. In the context of planar and cylindrically symmetric flow, a comprehensive analysis is conducted to examine the impact of radiation, mass fraction of solid particles (kp), ratio of specific heat of solid particles to specific heat of gas at constant pressure (β), axial magnetic field (μ), and non-idealness parameter on the length and velocity of sawtooth wave.

For the purpose of developing the small amplitude high frequency asymptotic solution for the system of nonlinear partial differential equations that describe one-dimensional compressible unsteady, planar, and non-planar flows in a dusty gas, the many time scales technique is used. Both the formation of the shock wave in a dusty gas flow and the investigation of the resonant interaction of waves are carried out with the help of the theory of weakly non-linear geometrical acoustics. It has been determined that the set of inviscid Berger's equations with a known kernel is what constitutes the transport equations for the wave amplitude along the rays for the flow of dusty gas. And last, the subject of shock waves in dusty gases is also discussed in this article.

**FUTURE SCOPE**

In this section, we discuss potential future research that will supplement the theme of the thesis. The scope of our investigation is restricted to a one-dimensional system of non-linear partial differential equations in the field of gasdynamics. On the other hand, this investigation might be broadened to include non-linear partial differential equations in gas dynamics that involve two or more dimensions. These are the primary subjects that might be the focus of research in the future. The following are some suggestions for extending the work that is provided in the thesis, which we could bring to your attention:

- Analytical investigation of the Riemann issue in many gas dynamics regimes inside a higher dimensional system of non-linear partial differential equations.
- The Differential Constraint Method's closed form solution to the Generalized Riemann Problem for the higher dimensional Euler's equation for a range of gas dynamic regimes. This approach may be used to find the precise solution to the Generalized Riemann Problem for two-phase flow in gas dynamics on behalf of applications of Diffuse Constraint approach.



- To talk about how elementary waves interact for the Riemann problem in a higher dimensional system of partial differential equations that are non-linear in distinct gas dynamics regimes.
- The Riemann issue in higher dimensions Euler's equation is studied numerically for different gas dynamic regimes utilizing different kinds of Riemann solvers.

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