# Application of Queueing System in Health Care

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## ABSTRACT

*Health care planning and administration may be enhanced by employing the many methods encompassed by operational research. O.R. is concerned with using analytical tools to improve decision-making. This study aims to examine the theory (Queuing) and examples (from across the world) of the use of queuing theory in healthcare organizations, as well as the advantages gained from doing so. Wait times for those seeking medical attention are unfortunately common. Waiting too long might result in annoyance, financial loss, and health problems down the road. This compels us to examine the theory of queues in further detail in order to learn about the factors that lead to the formation of queues and the guidelines for creating efficient systems. Queueing models shed light on the relationship between resource usage and variability. Increasing resource utilization reduces the per-patient cost of providing those services, but it might cause longer wait times for patients if there is uncertainty in either the service requirements or the number of service requests. Of fact, waiting times grow at an increasing rate in usage, indicating that the effect of resource utilization is extremely nonlinear at a certain degree of variability. This means that the cost of supplying resources and the cost of patient waiting time must be weighed in the capacity planning process in healthcare environments where considerable unpredictability is prevalent and difficult to remove. In this chapter, we analyse the value of queueing models to healthcare operations managers and review some of the fundamental models that assist quantify the aforementioned tradeoff. Specifically, we provide a brief summary of some previously established results for queueing systems that make use of a single server, several servers, a finite waiting area, a priority system, and a network of service points.*

## 1. INTRODUCTION

Now a days it became important to use queue models in many sectors as banks, airlines traffic, telecommunication network providing companies also the police services are also using queue models to response in time to the required people. Queue analysis is used in healthcare and other medical services, but has not gained popularity in this field. As we see that doctors are late, silence is common in hospitals and pharmacies. So, the healthcare industry is thinking about a queuing model that can be very convenient for patients and their family members who are waiting for an opportunity in the queue. This is important, so queuing analysis is also an important tool for estimating capacity conditions for possible unborn scripts, similar as demand peaks caused by new conditions or acts of terrorism. This chapter describes introductory queuing models that can be useful in health care and provides exemplifications of their use. Queuing proposition was developed byA.K. Erlang answered the capacity determination of the telephone system of the Danish telephone in 190( Brockmeyer etal. 19 8). Since also, it has been used to break numerous problems in the service assiduity, call centers( Brewton 1989, Stern and Hersh 1980, Holloran and Byrne 1986, Brusco etal. 1995, Brigandi etal. 199). line models aren't only used to identify applicable staffing, outfit, and bed situations, but also to resource allocation and the design of new services. Queuing proposition was developed byA.K. Erlang answered the capacity determination of the telephone system of the Danish telephone in 190( Brockmeyer etal. 19 8). Since also, it has been used to break a variety of problems in the service assiduity and telephone call centers( Brewton 1989, Stern and Hersh 1980, Holloran and Byrne 1986, Brusco etal. 1995, Brigandi etal. 1999).Queuing models are used not only to find suitable staff, equipment and beds, but also to allocate resources and plan new services. Less data is needed to analyze the situation and draw conclusions. These models are easier and cheaper to use. Because these models work quickly, they provide a very simple platform for what-if analysis, identifying trade-offs and taking very practical actions. We offer solutions. Technology-based development can be relatively expensive initially, as costs incurred during research and development are written off.

Unlocking the potential of these assets will likely take the next 5-7 years, and the tactics used can have a significant impact on both their effectiveness and their adoption rate. With the beginning of development and the explosive growth of information and communication technology, large-scale costs can be reduced significantly. For example, the cost of genome sequencing has dropped dramatically over the past 15 years. Emerging technologies are changing how clients access health care, how and which providers deliver health care, and health outcomes achieved through health care. His nine new technologies include robotic process automation, 3D printing, big data and analytics, artificial intelligence, targeted and personalized medicine, connected and cognitive devices, electrical devices and targeted and personalized devices. a list Some of these advances are unique to the healthcare industry, others are more advanced outside of healthcare, but have great potential for the industry. Using these new technologies Cases and value sources do not exist in isolation. Innovators think about how they can contribute and bring about dramatic change. Looking to the future of healthcare, there are four industry-level changes that could disrupt the current set of healthcare values A new, personal and intuitive health ecosystem. The most significant change may be the development of intuitive and personalized care ecosystems based on patients and their families and the integration of social and nursing networks.

## 2. REVIEW LITERATURE

**Goswami and N. Selveragu (2016)** discussed phase type arrivals and impatient customs in multi server queue. D.D.J.Q. and Soo Keng (2015) studied analysis of ticket queues with reneging customers. Janos Sztrik,et.al (2002), studied Simulation of machine interference in randomly changing environments.

**K.H. Wang, W.L. Chen, D.Y. Yang (2009)** dealt with the optimal control of machine repair problem with working holidays, they used Newton`s method X. Xu., Z. Zhang and N.Tian (2009) gave M/M/1 queue with single working holidays and arrangement times. Kharram, E(2008), analyzed an optional queuing model by dynamic number of repairman in finite population queuing system. J. C. Ke and K.H. Wang (2007), gave an idea of vacation policies of machine repair problem with two types spares.

**Lami, Haque and H.J. Armstrong (2007)** studied a survey of the machine interference. In (2006), S.P. Chen developed a mathematical programming approach to the machine interference problem with fuzzy parameters. Using spare parts and regressions, Madhu jain (2004) investigated the N approach for machine control systems, separately, use Alpha A.S. (2003) create holiday models. S.R. Chakraborty S. (2003) introduced phase type repairs and services and checked machine repair problem with unreliable server. M.J. Armstrong (2002) discussed lifetime repair guideline for the machine repair problem. A queue length distribution for nonvolatile computer was developed in 2000 by P.Patrick Wong. A. I. Soki worked omn M/C/KN in 2000 with bulking, withdrawal and past for a nested prototype of the machine. Budi Hardigo, I. and David, H.T. (1999) presented expected waiting time ranking in extended machine repair models. Gupta, S.M. (1997) performed the problem of machine interference with warm spares. C.H. Ng (1997) discussed fundamentals of queueing theory. Dshalalow J.H. (1997) developed queuing systems with state dependent parameters. A. N. Shirayaen, R.P. Boas's A.N.Shiriaer (1996) discussed the book entitled probability, E.E. Lewis (1196) presented introduction to reliability engineering. (417/600 Words ) English Paraphrased Text.

**R. Shivaraman et al. al (2017)** considered a queuing model for a machine repair system according to Bernoulli's holiday schedule. Downtime, repair time, and vacation time are assumed to follow an exponential distribution. The server can increase or decrease the queue length when congestion occurs. He modeled a finite-state Markov chain, whose stable distribution was obtained by a matrix recursion approach. C. Goswami and N. Selveragu (2016) discussed phase arrival and impatient habits in multiserver queues. D.D.J.Q Soo Keng (2015) studied the analysis of ticket queues with customer defection. Janos Sztrik, et.al (2002) studied simulations of machine interference in randomly varying environments.

**F.H. Miller, G.J.Leiberman (1995)** discussed some points in Operation Research. K,.H. Wang (1993) developed cost analysis of the M/M/R machines repair problems with mixed standby spares. J.Sztrik et.al (1993) analyzed the asymptotic behaviour of the machine interference problem with *N* machines and a single operative.

## 3. SERVICE TIME DISTRIBUTION IN THE HEALTH SECTOR

The term "patients" refers exclusively to outpatients (o.p.d.), namely student outpatients. In addition, the terms "customers" and "customers" are sometimes used interchangeably to describe people who visit a service point for a particular type of service. According to Iversen (1993), medical waiting lines are a sign of reduced efficiency when hospital resources are diverted from patient care. To increase the capacity of the emergency department (ED) to care for patients, Cochran et al. (2009) created an open queuing network model for ED scheduling. The technique is helpful for identifying hospital-specific variations in the patient acuity mix, arrival patterns, volumes, and process efficiency in a single shared computational model. The method is very helpful in determining hospital-specific patients arrival patterns, patient acuity mix, volumes and efficiencies of processes by a one common model. Bahadori et al. (201 ) discussed a descriptive method to study applied in a military hospital in Iran. He used a sample of 220 patients. The report of analysis reveals that the average numbers of patients arrived in the was reported on average 19.21 in the morning time and 1 .66 in evening time whereas average time spent by the patient or by his family member in the morning and evening were 39 minutes and 35 minutes respectively. The average system utilization were 25% and 21%. Armony et al. (2015) applied exploratory data analysis in a big Israeli hospital having important features which was not shown in the model. Green (2002) analysed data from New York State and predicted that number of bed unavailable in intensive care units (ICUs) and also in the obstetrics units by using the queuing means of queuing analysis. He found the number of units insufficient using different standards of patient delay. It concluded that around 0% of maternity wards and 90% of intensive care units did not have enough capacity to provide a suitable bed when needed.

## 4. ANALYSIS OF FIXED CAPACITY: NUMBER OF HOSPITAL BEDS?

It has been observed in many hospitals that they have limited resources, some resources have fixed capacity. They are working and thinking in the similar way they did in the past. They thinks about "things" rather than for people. They mainly think about beds, operation rooms, imaging machines (X-ray, Ultra sound), etcTo depict the utility of queuing model to figure out the capacity, we can consider an obstetric unit. As it's generally run singly from other services, its capacity conditions, similar as the number of postpartum beds, can be decided without taking other sanitarium areas into account. In addition, it's one where a typical M/ M/ s queuing paradigm works well. Studies of unscheduled sanitarium admissions have shown that the supposition of Poisson advents is a good bone because the maturity of obstetrics cases are unscheduled( youthful 1965). Queuing models aren't good enough for analysing similar coffers. In particular, if the cases staying for entering a service from a resource are handed fixed time places, there may be little or no liability of business unless cases visit late or the time places aren't large enough to accommodate utmost cases. A simple illustration of this would be the cases staying for a glamorous resonance imaging( MRI) installation which will be used by listed rehabilitants. still, there can be arise difficulty of managing numerous out-of-door cases when they visit at healthcare place demand for coffers is unscheduled and hence arbitrary, but it's important to watch those cases. In these cases, queuing models can be important tool in relating long- term capacity needs.

### 4.1 Application of Models

To depict the utility of queuing model to figure out the capacity, we can consider an obstetric unit. As it's generally run singly from other services, its capacity conditions, similar as the number of postpartum beds, can be decided without taking other sanitarium areas into account. In addition, it's one where a typical M/ M/ s queuing paradigm works well. Studies of unscheduled sanitarium admissions have shown that the supposition of Poisson advents is a good bone because the maturity of obstetrics cases are unscheduled ( youthful 1965). Additionally,

according to Green and Nguyen (2001), the CV for length of stay is often extremely close to 1.0, satisfying the M/M/s model's service time assumption.
The use of a queuing model might be either prescriptive or descriptive. We may use the arrival rate, average LOS, and bed count of a specific obstetrics unit as an illustration of the descriptive case and plug them into the specified equation to calculate the likelihood that a patient will not be able to locate a bed when they arrive. Assume that the obstetrics unit at town hospital has an average arrival rate of 15 patients per day, an average length of stay of 1/μ = 3 days, and 56 beds. The M/M/s calculation then yields an estimate of 4% using the likelihood of delay (1). We can use equation to utilize the M/M/s prescriptively to determine the minimal number of beds required to achieve a specific probability of delay.

**4.2 Assumptions for The Analysis**

1. Single waiting line will be served by single technical staff (1 server).
2. Patient's arrival pattern follow Poisson distribution
3. Time of service follows exponential distribution
4. Queue model followed is *FIFO* ( First in first out)
5. No patient will break the queue and will leave after getting served.

**4.3 Arrival Pattern Analysis**

The arrival pattern of patients/ relative was recorded in the five different centers for five days. Observed frequencies given in the Table 4.3 and expected frequency calculated using the formula

$$P(X=x)=\frac{\lambda^x e^{-\lambda}}{x!} \quad ;x=0,1,2,3,.... \qquad (1)$$

We will apply the Chi-square goodness of fit to test whether the model gives best fit or not.

**4.4 Analysis of Service Time**

The time taken to serve each person was recorded for 4 days and the recorded data are given in the Table 4.4. The distribution function (expected cumulative frequency) calculated applying the exponential distribution.
Exponential p.d.f. is given as

$$P(T=t)=\mu e^{-\mu t} \qquad (2)$$

$$for\ (t\geq 0) \qquad (3)$$

The distribution function for the service time when service time is less than t

$$P\ (service\ time\ <\ t)\ =1-e^{-\mu t} \qquad (4)$$

Again we require to use Chi –square goodness of fit to access goodness of fit.
On the basis of collected data mean arrival rate and service time is calculated. Then Queuing analysis is performed to find the expected number of patients in the queue as well as getting the service.
The expected number of patients/relatives waiting in the queue

$$L=\lambda * W \qquad (5)$$

Where L represents the number of persons who are waiting for their turn.
W signifies the waiting time (the remaining to get the service)
E(Patients in the queue)

$$Lq\ =\ \lambda Wq \qquad (6)$$

L represents customers count waiting in system, Wq is the waiting time in queue
E(Waiting time in the system)

$$W\ =\ Wq\ +\ 1/\lambda \qquad (7)$$

E(waiting time in queue)

$$Wq\ =\ \lambda/\left[\mu\ (\mu\ -\ \lambda)\right] \qquad (8)$$

For the singal server

$$P(zero\ patients\ waiting) = \rho^0 (1-\rho) \quad (9)$$

where $\rho = \lambda / \mu$.

$$P(n\ patients\ in\ the\ system) = \rho^n \Pr(0) \quad (10)$$

The average arrival rate calculated as 6.9 /minute

| k | Observed | | Fitted Poisson | |
|---|---|---|---|---|
| | Frequency | Proportion | Probability | Expected Frequency |
| 0 | 0 | 0 | 0.00112 | 0.621 |
| 1 | 6 | 0.0109 | 0.00764 | 4.2165 |
| 2 | 16 | 0.029 | 0.02593 | 14.3149 |
| 3 | 36 | 0.0652 | 0.05869 | 32.3994 |
| 4 | 54 | 0.0978 | 0.09963 | 54.9981 |
| 5 | 75 | 0.1359 | 0.1353 | 74.6874 |
| 6 | 75 | 0.1359 | 0.15312 | 84.5212 |
| 7 | 79 | 0.1431 | 0.14852 | 81.9856 |
| 8 | 62 | 0.1123 | 0.12606 | 69.5853 |
| 9 | 52 | 0.0942 | 0.09511 | 52.4982 |
| 10 | 30 | 0.0543 | 0.06458 | 35.6463 |
| 11 | 28 | 0.0507 | 0.03986 | 22.0035 |
| 12 | 19 | 0.0344 | 0.02255 | 12.4503 |
| 13 | 10 | 0.0181 | 0.01178 | 6.5029 |
| 14 | 6 | 0.0109 | 0.00571 | 3.1539 |
| 15 | 4 | 0.0072 | 0.00259 | 1.4277 |

**Figure:1**

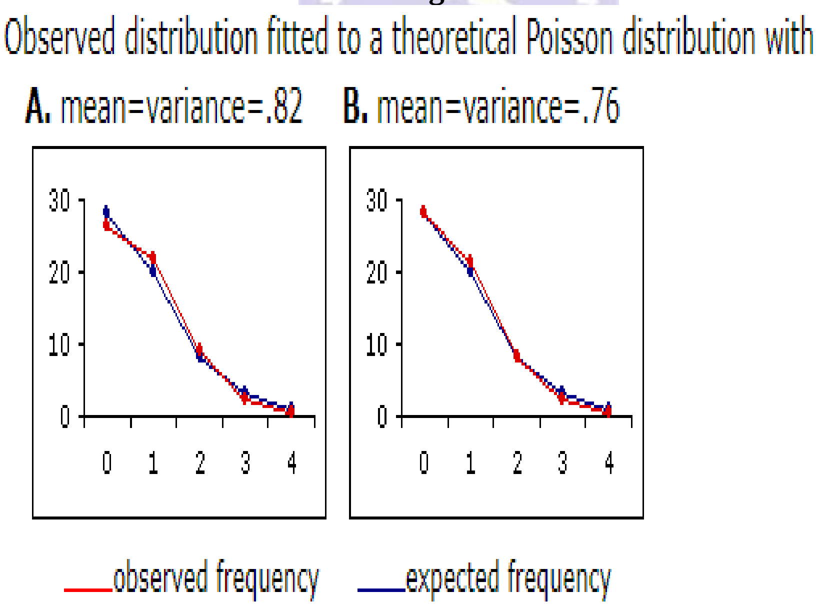


**Figure:2**

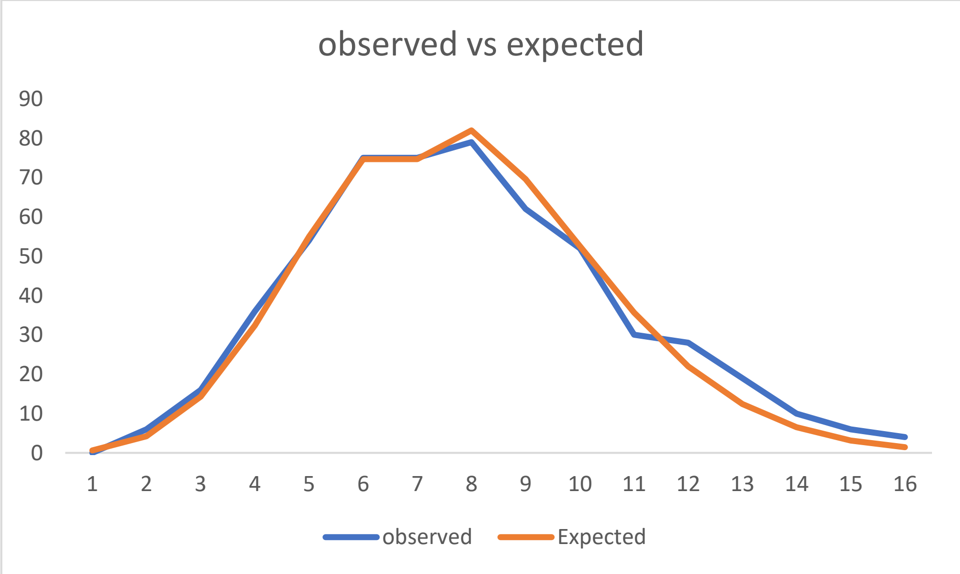


**Figure 3: Observed and expected frequencies**

To access the goodness of fit, we compute Chi-square test statistic

$$\chi^2 = \sum\left(\frac{O_i - E_i}{E_i}\right)^2 \tag{11}$$

Chi –square = 19.28
P –value = 0.200
Which tells fitting is good.

**Table :1**

| Arrival/per second (λ) | Service rate per second (μ) | L(Average customers in the system) |
|---|---|---|
| 2 | 4 | 1 |
| 2 | 6 | 0.5 |
| 4 | 6 | 2 |
| 6 | 12 | 1 |
| 8 | 14 | 1.33 |
| 10 | 14 | 2.5 |
| 16 | 18 | 8 |

**Table :2**

| Lq(Average customers in the queue) | W( Average waiting time) | Wq(Average time waiting in line |
|---|---|---|
| 0.5 | 0.5 | 0.25 |
| 0.17 | 0.25 | 0.08 |
| 1.33 | 0.5 | 0.33 |
| 0.5 | 0.17 | 0.08 |
| 0.76 | 0.17 | 0.1 |
| 1.79 | 0.25 | 0.18 |

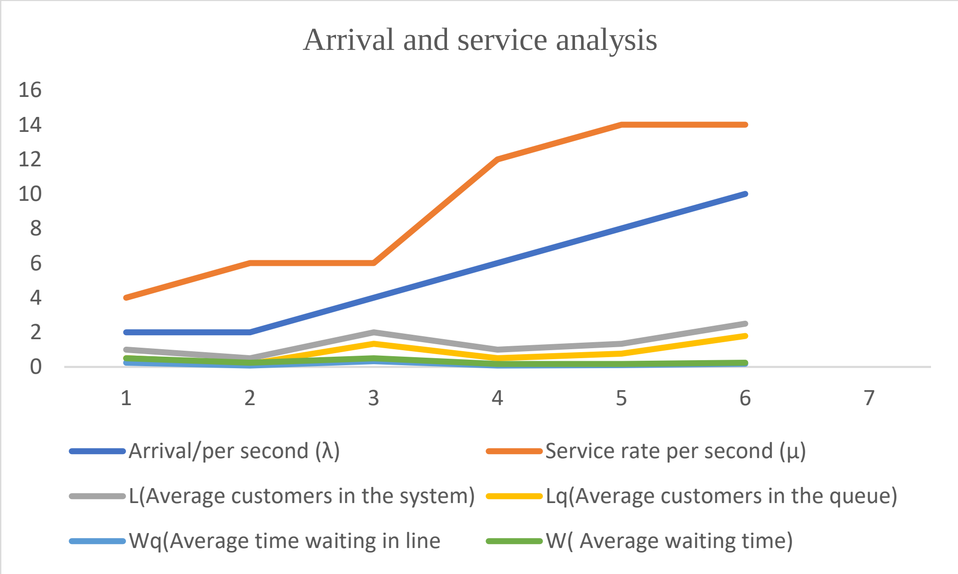


**Figure :4**

## 5. CONCLUSION

We have analyzed the data for the service time in the health sector by collecting the primary data. We used the M/M/S model to study the service discipline in the health sector. We have analyzed that as when the number of servers (doctors) is lesser and the patients count is low then waiting time for the patient in the queue is also low. As soon as the number of the patients increases the waiting time also increases. we have shown the application of M/M/S models in the Hospitals data for the patients waiting in the queue to get the service outside the operation theaters. We have discussed the advantages and disadvantages of the Monte Carlo Simulation. We concluded that if the servers available in the system are enough to provide the services to the customers the average length of the queue and the number of persons waiting inside and outside the system will be less. As soon as the number of the persons in the queue outside the system then the waiting time and the average time also increases. We also used simulation tool in the "R" software by using the library "queuing" and interpreted the result. The conditions were shown in the figures. The R code for the analysis has been provided in the Annexure.

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