Image Compression Through Global Thresholding Using Discrete Wavelet Transforms

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Abstract

Image compression is used to compress large files having two-dimensional signal into smaller files for efficiency of storage and transmission. Wavelet transform is used as an important tool for image compression. The compression of images is carried out by an encoder and outputs a compressed form of an image. Such a coder operates by transforming the data to remove redundancy, then quantizing the transform coefficients, and finally entropy coding the quantizer output. Thresholding is a technique to zero out wavelet coefficients below a particular value. The retained energy using biorthogonal wavelet is greater than that of Haar wavelet. Compression percentage is also greater in image compression for biorthogonal wavelet.

Keywords: Biorthogonal, Haar, image compression, global thresholding, retained energy, wavelet transforms

1. Introduction

In Fourier transform any signal or function is represented by superposition of sinusoidal components of different frequencies, therefore, it is widely used to analyse stationary signals/functions [1]. It has poor time frequency localization and less effective to analyse non-stationary and transient signals. In wavelet transforms, the signal is represented by superpositions of localized transient components called wavelets. It provides variable resolution in time and frequency domain both [2]. Wavelet transform captures local features and hence able to compress and denoise signals more efficiently [3]. Image is a two-dimensional signal perceived by the human visual system. A digital image is made up of pixels, which is an array of several pictures. Each pixel is denoted by a real number or a set of real numbers in a limited number of bits [4]. We can increase the storage and transmission process's performance by image compression. The image compression is implemented in software using MATLAB Wavelet Toolbox and 2-D DWT technique [5]. The experiments are carried out on .jpg format images.

There is a loss of some information in the process of image compression, which is acceptable. It is a key to rapid progress being made in information technology. Information can be represented in compact form by this process. For creating image file size transmittable, compression is an essential process. The main aim of this work is to investigate the compression of true colour image using wavelet theory. Image compression is of two types: lossless and lossy [6]. When we decrease the file size then we can store more images in each amount of memory space. Also, the time needed for images to be sent over the internet or downloaded from web pages reduces. The world has found itself between the large amounts of information with the growth of technology. Dealing with such large information can often present difficulties. Also, we need considerable storage capacity and transmission bandwidth for uncompressed graphics, video, and audio data. Wavelet compression is one of the ways to deal with this problem [7]. We explain the image compression through comparison of performance of discrete wavelets, such as Haar and biorthogonal wavelets.

2. Wavelet transforms

Wavelet is a small wave of zero average, which can be dilated and translated. A whole set of wavelets can be obtained by translating and scaling the mother wavelet as follows [8]: -

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}}\psi\left(\frac{x-b}{a}\right) = T_b D_a \psi$$

Here a and b are the dilation and translation parameters respectively.

Continuous wavelet transform of any function provides high amount of information about this function and can be expressed in terms of two parameters a and b as follows: -

$$W_{a,b} = \int f(x)\frac{1}{\sqrt{a}}\phi\left(\frac{x-b}{a}\right) dx$$
$$= \int f(x)\, \phi_{a,b}(x) dx$$

By introducing a = 2^{-j}, and $\frac{b}{a} = k$ where j and k are integers, the discrete wavelet transform is expressed as follows [9]: -

$$W_{j,k} = \int f(x) 2^{j/2}\, \phi(2^j x - k)\, dx$$

Here the function ϕ (x) is known as a scaling function of given MRA and can be expressed by a dilation equation as follows [10]: -

$$\phi(x) = \sum_{k\in\mathbb{Z}} \alpha_k\, \phi(2x - k)$$

where α_k is called a low pass filter and described as follows: -

$$\alpha_k = \left(\frac{1}{\sqrt{2}}\right) \int_{-\infty}^{\infty} \phi(x)\phi(2x - k)dx$$

If $\psi \in W_0$ be any function then,

$$\psi(x) = \sum_{k\in Z} \beta_k\, \phi(2x - k)$$

Where, $\beta_k = (-1)^{k+1}\, \alpha_{1-k}$ is high pass filter.

Any signal in vector space V_0 can be expressed in terms of bases of subspaces V_1 and W_1. Therefore, we can express space V_0 in terms of subspaces V_1 and W_1 as follows: -

$$V_0 = V_1 \oplus W_1$$

In general,

$$V_j = V_{j+1} \oplus W_{j+1}$$

But,

$$V_{j+1} = V_{j+2} \oplus W_{j+2}$$

Therefore,

$$V_j = W_{j+1} \oplus W_{j+2} \oplus V_{j+2}$$

$$V_j = W_{j+1} \oplus W_{j+2} \oplus W_{j+3} \oplus \ldots\ldots\ldots\ldots W_{j_0} \oplus V_{j_0}$$

Any integrable function f, has the series expansion,

$$f(x) = \sum_{k\in\mathbb{Z}} \langle f, \tilde{\phi}_{j_0,k} \rangle\, \phi_{j_0,k}(x) + \sum_{p=j+1}^{j_0} \sum_{k\in\mathbb{Z}} \langle f, \tilde{\psi}_{p,k} \rangle\, \psi_{p,k}(x)$$

Similarly, the roles of the basis and the dual basis can be interchanged [11]. Here, $\sum_{k\in\mathbb{Z}} \langle f, \tilde{\phi}_{j_0,k} \rangle\, \phi_{j_0,k}(x)$ is a coarse scale V_{j_0} – approximation of f and for every p, the sum $\sum_{k\in\mathbb{Z}} \langle f, \tilde{\psi}_{p,k} \rangle\, \psi_{p,k}(x)$, adds the detail in spaces W_p.

A digitization is a process by which any analogue image in continuous space can be converted into a digital image in discrete space through sample process. Let us consider that an analogue image can be divided into M rows and N columns. The value can be assigned to the integer coordinates [m, n] like {m = 0, 1, 2, . . ., M −1} and {n = 0, 1, 2, . . ., N − 1}. In image wavelet transforms, the scaling and wavelet function are described as two variable functions $\phi(x, y)$ and $\psi(x, y)$ [12-13]. The image scaling and wavelet functions are easily expressed in terms of 1D functions as follows: -

$$\phi(x, y) = \phi(x)\phi(y)$$
$$\psi^1(x, y) = \psi(x)\phi(y)$$
$$\psi^2(x, y) = \phi(x)\psi(y)$$
$$\psi^3(x, y) = \psi(x)\psi(y)$$

Obviously the 2D (Image) functions are described as the superposition of 1D scaling and wavelet functions. The 2D approximation and detail coefficients are determined as follows: -

$$a[j_0, m, n] = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)\, \phi_{j_0,m,n}(x, y)$$

$$d^i(j, m, n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)\, \psi^i_{j,m,n}(x, y)$$

where i = {1,2,3}

$$f(x, y) = \frac{1}{\sqrt{MN}} \sum_m \sum_n a[j_0, m, n]\, \phi_{j_0,m,n}(x, y) + \frac{1}{\sqrt{MN}} \sum_{i=1,2,3} \sum_{p=j+1}^{j_0} \sum_m \sum_n d^i(p, m, n)\, \psi^i_{p,m,n}(x, y)$$

In 2D-wavelet transforms of an image, the scaling and wavelet functions can be expressed as follows [14]: -

$$\phi_{j,m,n}(x, y) = 2^{j/2} \phi(2^j x - m, 2^j y - n)$$
$$\psi^i_{j,m,n}(x, y) = 2^{j/2} \psi^i(2^j x - m, 2^j y - n)$$

for $1 \le i \le 3$. The wavelet functions $\left\{\psi^1_{j,m,n}, \psi^2_{j,m,n}\, \psi^3_{j,m,n}\right\}$ form an orthonormal basis of the subspace of details and described as follows: -

$$W_j^2 = (V_j \otimes W_j)(W_j \otimes V_j) \oplus ((W_j \otimes W_j)$$

at scale j. The Lebesgue space $L^2(\mathbb{R}^2)$ can be expressed as follows: -

$$L^2(\mathbb{R}^2) = \sum_j W_j^2$$

3. Haar and biorthogonal wavelets

Haar wavelet resembles a step function and it is discontinuous. The Haar wavelet is the same as Daubechies1 [15]. The mother wavelet function $\psi(t)$ of Haar wavelet can be defined as:

$$\psi(t) = \begin{cases} 1 & 0 \leq t < 1/2 \\ -1 & 1/2 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

Its $\phi(t)$ can be defined as: -

$$\phi(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

The energy of signal and compaction of the energy of signals conserved by Haar transform. The Haar transform has been commonly used for image processing and pattern recognition, due to its low computing requirement. Also, Haar transform can be used for removing noise and compressing audio signals. A discrete signal is decomposed into two sub-signals of half of its length by Haar transform. One sub-signal is a running average and another sub-signal is a running difference. Haar wavelets are related to the mathematical operator, which is called Haar transform in discrete form. It shows orthogonal, biorthogonal and compact support. Its DWT as well as CWT is also possible. Its scaling and wavelet function are shown in figure 1 and 2.

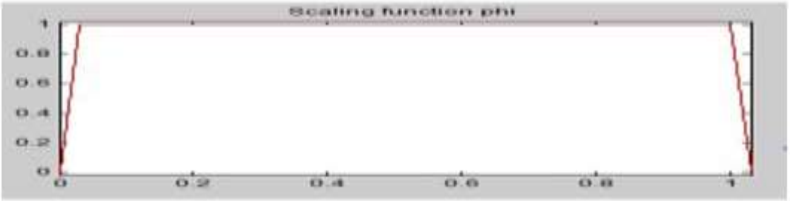


Figure 1: Scaling function of Haar wavelet

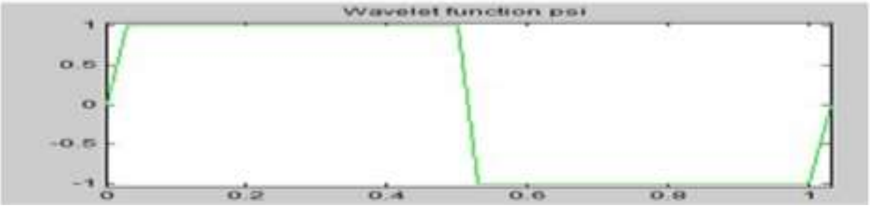


Figure 2: Wavelet function of Haar wavelet

Biorthogonal wavelet shows the property of linear phase. Biorthogonal wavelets can be used to decompose and recover function. Its linear phase property is useful for signal and image construction. The wavelet transform associated with a biorthogonal wavelet is invertible but not necessarily orthogonal [16]. The decomposition scaling and wavelet function is shown in figure 3 and 4.

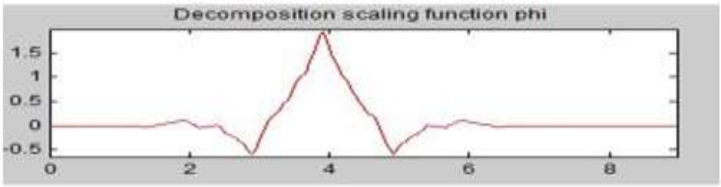


Figure 3: Scaling function of biorthogonal wavelet

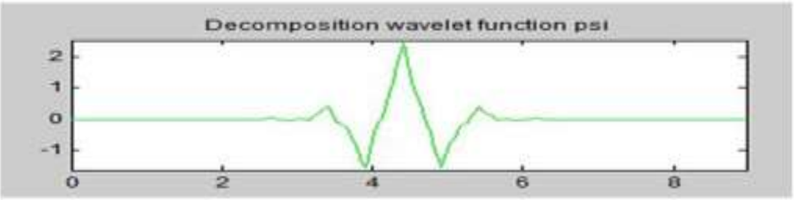


Figure 4: Wavelet function of biorthogonal wavelet

Before starting the processing of larger images or videos, compression is an important step in the field of Image processing. The compression of images is carried out by an encoder and outputs a compressed form of an image. The mathematical transforms play a vital role in the processes of compression [17]. The process of the compression of images can be represented in flow chart which is given as:

4. Image compression

Image compression is one of the most useful and commercially successful technologies in the field of image processing. It is necessary to handle large amounts of information such as multimedia. We also need image compression to fulfil the goal of representing an image with a minimum number of bits. Image compression is also needed for focusing on reduction of several types of redundancy in data or information. Compression plays an important role in creating file sizes of manageable and transmittable dimensions. It is another method which is increasing the bandwidth, but it is a less attractive solution due to its cost. It is used to reduce the binary representation (e.g., file size) of images. So, within a fixed amount of disc space you can store more images. Smaller images are also easier to transfer, e-mail, etc. By shrinking the size of the image, the value of the pixel becomes smaller, needs to be stored and consequently the file will take less time to load. Image compression can increase the efficiency of processing, recording and storage in the security industry. For any organization such as a federal government agency, image compression is also useful which requires the storing of images to be standardized. In museums, compressed images can be used for the education of that establishment's visitors. Without putting a large load on the server, information can be stored with the help of image compression. Image compression is of great importance in digital image processing because of their wide application.

Wavelet transforms have the great advantage of being capable of separating the fine details of a signal. Wavelet based coding provides substantial improvements in picture quality at higher compression ratios. Wavelet transform is widely used for image compression [18]. Many powerful and artificial wavelet-based schemes for image compression have been developed and implemented over the past few years. Wavelet based compression algorithms are suitable for the new JPEG-2000 standard. Such a coder operates by transforming the data to remove redundancy, then quantizing the transform coefficients (a lossy step), and finally entropy coding the quantizer output [19].

4.1 Global thresholding

The process of separating some regions (or their counters) in image processing is called segmentation. So, the natural way to segment such regions is called thresholding [20]. It is nothing but separations of dark and light regions. Any gray scale image can be converted into a binary image using thresholding by setting some threshold value. Below the threshold value the function has an expression while above the value it has another. Singular value thresholding can be given mathematically as follows: -

$$g(x, y) = \begin{cases} 1 & \text{if } f(x, y) > T \\ 0 & \text{if } f(x, y) \leq T \end{cases}$$

Global thresholding means a single value of pixel intensity threshold is used for all pixels in the images for converting it into a binary image. The properties of pixel and grey level value of the image are responsible for threshold value T. A single threshold for all the image pixels is used in global thresholding [21].

4.2 Process of image compression

The process of image compression using wavelet transforms is as follows: -

Figure 5: Block diagram of image compression processes

4. Methodology

We open the wavelet toolbox of the main menu of MATLAB and do the following steps to compress our image: -

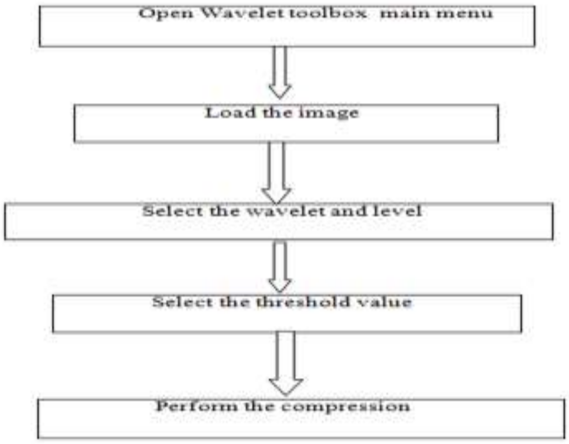


Figure 6: Methodology for image compression

For image compression using wavelet transforms, we have used MATLAB software. We have selected image of a baby as the original or raw image.

Figure 7: Original image

Figure 8: Compressed Image using Haar and biorthogonal wavelet transforms

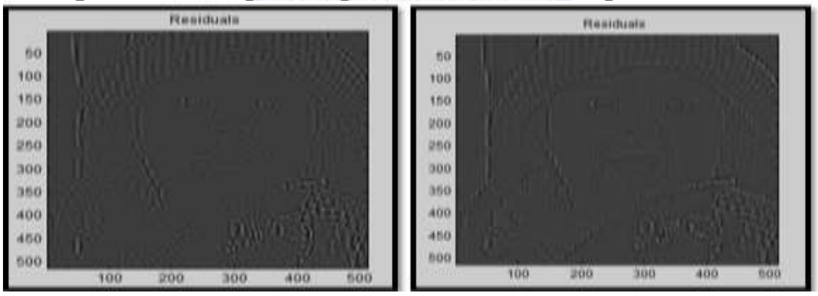


Figure 9: Residuals of image

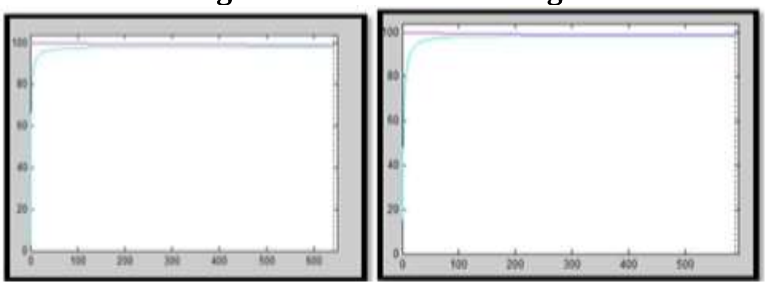
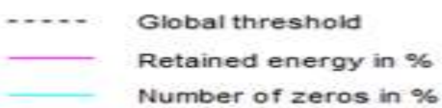


Figure 10: Retained energy



6. Results and discussion

The Haar and Biorthogonal wavelet transform of a baby image are performed with help of given methodology. The results obtained are listed in table 1.

Table 1: Experimental results

S. No.	Wavelet	Size of original image in bytes	Size of compressed image in bytes	Compression %
1.	Haar	44,825	27,226	38.14%
2.	Biorthogonal	44,825	25,015	44.19%

The residual of image is obtained by deducting the denoised image from noisy image. It discloses the extreme smoothing and blurring of small details exists in an image. Residuals of the compressed image using Haar and biorthogonal wavelet transforms are shown in figure 9. While the retained energy of compressed image are shown in figure 10. We compare the performance of these transforms on image Baby.jpg (512x512). More percentage of zeros shows more compression and due to higher value of retained energy, less information loss is observed. In Haar wavelet retained energy is 98.92% and in biorthogonal transform it is 99.18%. The compression percentage of image is found more for biorthogonal transform than that of Haar wavelet transform.

7. Conclusion

Wavelet transform is a suitable tool for image processing. The results of image compression using wavelet transforms are very nice and fruitful. A baby image is taken as an original image and wavelet transform is performed using Haar and biorthogonal wavelet. The retained energy of the compressed image represents to the compression ratio of the image. The results using Haar and biorthogonal wavelet transforms are discussed and compared. The wavelet transforms provide a simple and accurate framework for the modelling the behaviour of image compression and processing.

References

1. Strang, G.,1993. Wavelet transforms versus Fourier transforms, Bulletin of the American Mathematical Society, Vol. 28, Issue 2, 288-305.
2. Hernandez, E. and Weiss, G. 1996. A First Course on Wavelets. CRC Press, New York.
3. Antoine, J. P., 2004. Wavelet analysis: A new tool in Physics. Wavelets in Physics, Van Den Berg, J. C., ed., 9-21.
4. Mathur, S., Purohit, R. and Vyas, A., 2016. A review on basics of digital image processing, International Journal of Engineering Research & Technology, Special Issue, 1-3.
5. Pratap, R., 2006. Getting started with MATLAB, 2nd Edition 2006, 1-15, 17-44 & 49-79.
6. Hiremath, S. and Rani,S., 2020. A Concise Report on Image Types, Image file format and noise model for image processing, International Research Journal of Engineering and Technology, Vol. 7, Issue 8, 4991-4997.
7. Kumar, B.B.S. and Satyanarayana, P.S., 2013. Image analysis using biorthogonal wavelet, International Journal of Innovative Research and Development, Vol. 2, Issue 6, June 2013, 543-565.
8. Heil, C., Walnut, D., 1989. Continuous and discrete wavelet transforms, SIAM review, Vol. 31, 628-666.
9. Goswami, J. C., and Chan, A.K. 1999. Fundamentals of Wavelets Theory, Algorithms and Applications. Willey-Interscience, New York.
10. Faizer, M. and Jawerth, B., 1990. A discrete transform and decomposition of distribution spaces, Journal of Functional Analysis, Vol. 93, Issue 2, 134-170.
11. Mallat, S.G., 1989. Multiresolution approximations and wavelet orthonormal bases of, Transactions of the American Mathematical Society, Vol. 315, 69 – 87.
12. Wallace, G. K., 1991. The JPEG still picture compression standard, Comm. ACM, Vol. 34, Issue 4, 30-44.
13. Said, A. and Pearlman, W.A.,1996. An image multiresolution representation for lossless and lossy compression, IEEE Trans. on Image Processing, Vol. 5, Issue 9, 1303-1310.

14. Gonzalez, R.C., Woods, R.E. and Eddins, S.L., 2006. Digital Image Processing using MATLAB, 1st Edition 2006, 1- 78, 256-295 & 296-547.
15. Daubechies, I., 1990. The wavelet transforms, time frequency localization and signal analysis, IEEE Transaction on Information Theory, Vol. 6, 961 – 1005.
16. Mallat, S.G., 1989. Multiresolution approximations and wavelet orthonormal bases of $L^2(R)$, Transactions of the American Mathematical Society, Vol. 315, 69-87.
17. Alarcon-Aquino, V., Ramirez-Cortes, J. M., Gomez-Gil, P., Starostenko, O. and Lobato-Morales, H., 2013. Lossy image compression using discrete wavelet transform and thresholding techniques, The Open Cybernetics &Systemics Journal, Vol. 7, 32-38.
18. Kharate, G. K., Patil, V. H. and Bhale, N. L., 2007. Selection of mother wavelet for image compression on basis of nature of image, Journal of multimedia, Vol. 2, Issue 6, 44-51.
19. Surabhi, N., Unnithan, S.N., Image compression techniques: A review, International Journal of Engineering Development and Research, Vol. 5, Issue 1, 585-589.
20. Simhadri, K. K., Iyengar, S. S., Holyer, R.J., Lybanon, M. and Zachary, J.M., 1998. Wavelet-based feature extraction from oceanographic images, IEEE Trans. Geoscience and Remote Sensing, Vol. 36, Issue 3, 767-778.
21. Yannan, W., Shudong, Z. and Hui, L., 2013. Study of image compression based on wavelet transform, Proc. IEEE- Fourth Int Conf on Intelligent Systems Design and Engineering Applications, 575-578.