

PROBABILITY DISTRIBUTION OF INTEGRAL INVOLVING HYPERGEOMETRIC FUNCTION

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4.1 MAIN INTEGRALS

In this section, the following probability distribution of thirty nine integrals involving hypergeometric functions have been obtained in the form of a single integral.

$$\int_0^1 x^{c-1} (1-x)^{c-e+1} [1 + \alpha x + \beta(1-x)]^{-2c+e-i-1} F_1^2(a, 1+j-a, e; \frac{(1+\alpha)x}{1+\alpha x + \beta(1-x)}) dx =$$

$$\frac{\Gamma(e)\Gamma(c-e+i+1)\Gamma(e-c-\frac{1}{2}(i+|i|))\Gamma(c-\frac{1}{2}(j+|j|))\Gamma(a-\frac{1}{2}(i+j+|i+j|))}{2^{2a-i-j}(1+\alpha)^c(1+\beta)^{c-e+i+1}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+i+1)} x \{D_{i,j}$$

$$\frac{\Gamma(c+\frac{1}{2i}-\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2})\Gamma(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{4}(1+(-1)^i))}{\Gamma(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}+\frac{(-1)^i}{4})((-1)^i-1+[-j/2])\Gamma(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2i}+[-\frac{j}{2}])}$$

$$E_{i,j} \frac{\Gamma(c+\frac{1}{2i}-\frac{1}{2e}-\frac{1}{2a}+1)\Gamma(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{4}(1+(-1)^i))}{\Gamma(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}+\frac{(-1)^j}{4})((-1-(-1)^i)+[-\frac{j}{2}+\frac{1}{2}])\Gamma(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2i}-\frac{1}{2}+[-\frac{j}{2}+\frac{1}{2}])} \dots\dots\dots(4.1.1)$$

for $i, j = 0, \pm 1, \pm 2, \pm 3$.

Also, provided $\text{Re}(e) > 0, \text{Re}(c-e+i+1) > 0$ for $i = 0, \pm 1, \pm 2, \pm 3$ and $\text{Re}(c) > j$ for $j = 1, 2, 3$. also the constants α and β are such that no one of the expressions $1+\alpha, 1+\beta$ and $1+\alpha x + \beta(1-x)$, where $0 \leq x \leq 1$, is zero. Again, as usual $[x]$ is the greatest integer less than or equal to x . the coefficient $D_{i,j}$ and $E_{i,j}$ are given in the tabular form.

So by well known definition of probability distributions we have:

$$F(x) = \frac{\Gamma(e)\Gamma(c-e+i+1)\Gamma(e-c-\frac{1}{2}(i+|i|))\Gamma(c-\frac{1}{2}(j+|j|))\Gamma(a-\frac{1}{2}(i+j+|i+j|))}{2^{2a-i-j}(1+\alpha)^c(1+\beta)^{c-e+i+1}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+i+1)} x$$

$$\{D_{i,j} \frac{\Gamma(c+\frac{1}{2i}-\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2})\Gamma(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{4}(1+(-1)^i))}{\Gamma(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}+\frac{(-1)^i}{4})((-1)^i-1+[-j/2])\Gamma(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2i}+[-\frac{j}{2}])}$$

$$E_{i,j} \frac{\Gamma(c+\frac{1}{2i}-\frac{1}{2e}-\frac{1}{2a}+1)\Gamma(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{4}(1+(-1)^i))}{\Gamma(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}+\frac{(-1)^j}{4})((-1-(-1)^i)+[-\frac{j}{2}+\frac{1}{2}])\Gamma(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2i}-\frac{1}{2}+[-\frac{j}{2}+\frac{1}{2}])}\}$$

$$\int_0^1 x^{c-1} (1-x)^{c-e+1} [1 + \alpha x + \beta(1-x)]^{-2c+e-i-1} dx$$

=0, elsewhere

$$=1, \int_0^1 f(x) dx = 1$$

Where $f(x) = F_1^2(a, 1+j-a, e; \frac{(1+\alpha)x}{1+\alpha x + \beta(1-x)})$

4.2 SPECIAL CASES

1. If we set $I, j = 0, \pm 1, \pm 2$, in (4.2.1), we get twenty five integrals obtained earlier by Nagar [108] and gaur(2003).

2. On the other hand, fourteen integrals for different values of I and j other than obtained by Nagar[108] and gaur(2003).

First Formula

$$\int_0^1 x^{c-1} (1-x)^{c-e+3} [1 + \alpha x + \beta(1-x)]^{-2c+e-4} F_1^2(a, 1-a, e; \frac{(1+\alpha)x}{1+\alpha x + \beta(1-x)}) dx =$$

$$\frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3)\Gamma(c)}{2^{2a}(1+\alpha)^c(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(2c-e-a+4)} x \{ \{-(a+2)(a-3)+3c(c+3)-e(3c-$$

$$e+5)\} \frac{\Gamma(c-\frac{a}{2}-\frac{e}{2}+2)\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{1}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+2)} + \{(a+1)(a-2)-c(c+3)-e(c-$$

$$e+3)\} \frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+\frac{5}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2})} \dots\dots\dots(4.2.1)$$

Provided $\text{Re}(c) > 0, \text{Re}(c-e+4) > 0, \text{Re}(e) > 0$. Also the constants α and β are such that none of the expressions $1+\alpha, 1+\beta$ and $1+\alpha x+\beta(1-x), 0 \leq x \leq 1$, is not zero.

So by well known definition of probability distributions we have:

$$F(x) = \frac{\frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3)\Gamma(c)}{2^{2a}(1+\alpha)^c(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+4)} x^{\frac{\Gamma(c-\frac{a}{2}-\frac{e}{2}+2)\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+2)}} \left[\{-(a+2)(a-3)+3c(c+3)-e(3c-e+5)\} \right. \\ \left. + \{(a+1)(a-2)-c(c+3)-e(c-e+3)\} \frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+\frac{5}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2})} \right] \\ \int_0^1 x^{c-1} (1-x)^{c-e+i} [1+\alpha x+\beta(1-x)]^{-2c+e-4} dx$$

=0, elsewhere

$$=1, \int_0^1 f(x) dx = 1$$

Where $f(x) = F_1^2(a, 1-a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Second Formula

$$\int_0^1 x^{c-1} (1-x)^{c-e+3} [1+\alpha x+\beta(1-x)]^{-2c+e-3} F_1^2(a, -a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx = \\ \frac{\Gamma(e)\Gamma(c-e+3)\Gamma(e-c-2)\Gamma(c)}{2^{2a+1}(1+\alpha)^c(1+\beta)^{c-e+3}\Gamma(e-a)\Gamma(e-c)\Gamma(2c-e-a+3)} x^{\frac{\Gamma(c-\frac{a}{2}-\frac{e}{2}+\frac{3}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2})}} \left[\{-(a+1)(a-2)+c(a+c+3)-e(2c-e+3)\} \right. \\ \left. + \{(a-1)(a+2)+c(a-c+3)-e(2c-e+3)\} \frac{\Gamma(c+2-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+2)} \right] \dots \dots \dots (4.2.2)$$

Provided $\text{Re}(c) > 0, \text{Re}(c-e+4) > 0, \text{Re}(e) > 0$. Also the constants α and β are such that none of the expressions $1+\alpha, 1+\beta$ and $1+\alpha x+\beta(1-x), 0 \leq x \leq 1$, is not zero.

So by well known definition of probability distributions we have:

$$F(x) = \frac{\frac{\Gamma(e)\Gamma(c-e+3)\Gamma(e-c-2)\Gamma(c)}{2^{2a+1}(1+\alpha)^c(1+\beta)^{c-e+3}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+3)} x^{\frac{\Gamma(c-\frac{a}{2}-\frac{e}{2}+\frac{3}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2})}} \left[\{-(a+1)(a-2)+c(a+c+3)-e(2c-e+3)\} \right. \\ \left. + \{(a-1)(a+2)+c(a-c+3)-e(2c-e+3)\} \frac{\Gamma(c+2-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+2)} \right] \\ \int_0^1 x^{c-1} (1-x)^{c-e+i} [1+\alpha x+\beta(1-x)]^{-2c+e-4} dx$$

=0, elsewhere

$$=1, \int_0^1 f(x) dx = 1$$

Where $f(x) = F_1^2(a, -a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Third Formula

$$\int_0^1 x^{c-1} (1-x)^{c-e+1} [1+\alpha x+\beta(1-x)]^{-2c+e-2} F_1^2(a, 1-a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx = \\ \frac{\Gamma(e)\Gamma(c-e+2)\Gamma(e-c-1)\Gamma(c)}{2^{2a+2}(1+\alpha)^c(1+\beta)^{c-e+2}\Gamma(e-a)\Gamma(e-c)\Gamma(2c-e-a+2)} x^{\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+1)\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+2)}} \left[\{-(a+1)(a-2)-2c(c+2)-e(3c-e+3)\} \right. \\ \left. + \{-a(a+1)-e(c-e+1)\} \frac{\Gamma(c+\frac{3}{2}-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2})} \right] \dots \dots \dots (4.2.3)$$

Provided $\text{Re}(c) > 0, \text{Re}(c-e+2) > 0, \text{Re}(e) > 0$. Also the constants α and β are such that none of the expressions $1+\alpha, 1+\beta$ and $1+\alpha x+\beta(1-x), 0 \leq x \leq 1$, is not zero.

So by well known definition of probability distributions we have:

$$F(x) = \frac{\frac{\Gamma(e)\Gamma(c-e+2)\Gamma(e-c-1)\Gamma(c)}{2^{2a+2}(1+\alpha)^c(1+\beta)^{c-e+2}\Gamma(e-a)\Gamma(e-c)\Gamma(2c-e-a+2)} x^{\frac{\Gamma(c-\frac{e-a}{2}-\frac{a}{2}+1)\Gamma(\frac{e-a}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}+\frac{1}{2})-\Gamma(c-\frac{e-a}{2}+\frac{a}{2}+2)}} + \frac{\{(a-1)(a+2)-2c(c+2)-e(3c-e+3)\}}{\Gamma(\frac{e-a}{2}+\frac{1}{2})-\Gamma(c-\frac{e-a}{2}+\frac{a}{2}+2)} + \frac{\{-a(a+1)-e(c-e+1)\}}{\Gamma(\frac{e-a}{2}+\frac{1}{2})\Gamma(c-\frac{e-a}{2}+\frac{a}{2}+\frac{3}{2})} \frac{\Gamma(c+\frac{3}{2}-\frac{e-a}{2}-\frac{a}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2}+\frac{1}{2})\Gamma(c-\frac{e-a}{2}+\frac{a}{2}+\frac{3}{2})}]$$

$$F(x) = \int_0^1 x^{c-1} (1-x)^{c-e+i} [1+\alpha x+\beta(1-x)]^{-2c+e-4} dx$$

=0, elsewhere

=1, $\int_0^1 f(x) dx = 1$

Where $f(x) = F_1^2(a, 1-a; e; \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Fourth Formula

$$\int_0^1 x^{c-1} (1-x)^{c-e} [1+\alpha x+\beta(1-x)]^{-2c+e-1} F_1^2(a, -2-a, e; \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx =$$

$$\frac{\Gamma(e)\Gamma(c-e+1)\Gamma(c)}{2^{2a+3}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(e-a)\Gamma(2c-e-a+1)} x^{\frac{\Gamma(c+\frac{1}{2}-\frac{e-a}{2}-\frac{a}{2})\Gamma(\frac{e-a}{2}-\frac{a}{2})}{\Gamma(\frac{e-a}{2}+\frac{1}{2})-\Gamma(c-\frac{e-a}{2}+\frac{1}{2}+\frac{3}{2})}} + \frac{\{e(2c-e+1)+a(a-c+1)\}}{\Gamma(\frac{e-a}{2}+\frac{1}{2})-\Gamma(c-\frac{e-a}{2}+\frac{1}{2}+\frac{3}{2})} + \frac{\{e(2c-e+1)+a(a+2)(a+c+1)\}}{\Gamma(\frac{e-a}{2}+\frac{3}{2})\Gamma(c-\frac{e-a}{2}+\frac{a}{2}+2)} \frac{\Gamma(c+1-\frac{e-a}{2}-\frac{a}{2})\Gamma(\frac{e-a}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}+\frac{3}{2})\Gamma(c-\frac{e-a}{2}+\frac{a}{2}+2)} \dots \dots \dots (4.2.4)$$

Provided $\text{Re}(c) > 0, \text{Re}(c-e+1) > 0, \text{Re}(e) > 0$. Also the constants α and β are such that none of the expressions $1+\alpha, 1+\beta$ and $1+\alpha x+\beta(1-x), 0 \leq x \leq 1$, is not zero.

So by well known definition of probability distributions we have:

$$F(x) = \frac{\frac{\Gamma(e)\Gamma(c-e+1)\Gamma(c)}{2^{2a+3}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(e-a)\Gamma(2c-e-a+1)} x^{\frac{\Gamma(c+\frac{1}{2}-\frac{e-a}{2}-\frac{a}{2})\Gamma(\frac{e-a}{2}-\frac{a}{2})}{\Gamma(\frac{e-a}{2}+\frac{1}{2})-\Gamma(c-\frac{e-a}{2}+\frac{1}{2}+\frac{3}{2})}} + \frac{\{e(2c-e+1)+a(a-c+1)\}}{\Gamma(\frac{e-a}{2}+\frac{1}{2})-\Gamma(c-\frac{e-a}{2}+\frac{1}{2}+\frac{3}{2})} + \frac{\{e(2c-e+1)+a(a+2)(a+c+1)\}}{\Gamma(\frac{e-a}{2}+\frac{3}{2})\Gamma(c-\frac{e-a}{2}+\frac{a}{2}+2)} \frac{\Gamma(c+1-\frac{e-a}{2}-\frac{a}{2})\Gamma(\frac{e-a}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}+\frac{3}{2})\Gamma(c-\frac{e-a}{2}+\frac{a}{2}+2)}]$$

$$F(x) = \int_0^1 x^{c-1} (1-x)^{c-e+i} [1+\alpha x+\beta(1-x)]^{-2c+e-4} dx$$

=0, elsewhere

=1, $\int_0^1 f(x) dx = 1$

Where $f(x) = F_1^2(a, -2-a; e; \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Fifth Formula

$$\int_0^1 x^{c-1} (1-x)^{c-e+3} [1+\alpha x+\beta(1-x)]^{-2c+e-4} F_1^2(a, 2-a, e; \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx =$$

$$\frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3)\Gamma(c)\Gamma(a-1)}{2^{2a+1}(1+\alpha)^c(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+4)} x^{\frac{\Gamma(-a(a-1)(a+e-1))}{\dots}}$$

$$3)+c(a+c)\left\{\frac{\Gamma(c+2-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{1}{2})-\Gamma(c-\frac{e}{2}+\frac{a}{2}+1)}+\{(a-1)(a-e+1)+c(a-c-2)\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+\frac{5}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-1)\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2})}\right\}\dots\dots\dots(4.2.5)$$

Provided $\text{Re}(c)>0, \text{Re}(c-e+4)>0, \text{Re}(e)>0$. Also the constants α and β are such that none of the expressions $1+\alpha, 1+\beta$ and $1+\alpha x+\beta(1-x), 0 \leq x \leq 1$, is not zero.

So by well known definition of probability distributions we have:

$$F(x)=\frac{\left[\frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3)\Gamma(c)\Gamma(a-1)}{2^{2a+1}(1+\alpha)^c(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+4)}\right]^x \left\{-a(a-1)(a+e-3)+c(a+c)\right\}\frac{\Gamma(c+2-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{1}{2})-\Gamma(c-\frac{e}{2}+\frac{a}{2}+1)}+\{(a-1)(a-e+1)+c(a-c-2)\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+\frac{5}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-1)\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{3}{2})}}{\int_0^1 x^{c-1} (1-x)^{c-e+4} [1+\alpha x+\beta(1-x)]^{-2c+e-4} dx}$$

=0, elsewhere
=1, $\int_0^1 f(x)dx = 1$

Where $f(x) = F_1^2(a, -a; e; \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Sixth Formula

$$\int_0^1 x^{c-1} (1-x)^{c-e+3} [1+\alpha x+\beta(1-x)]^{-2c+e-4} F_1^2(a, 3-a; e; \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx = \frac{\left[\frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3)\Gamma(c)\Gamma(a-2)}{2^{2a+2}(1+\alpha)^c(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+4)}\right]^x \left\{-(a-1)(a-2)+c(2c-e+2)\right\}\frac{\Gamma(c+2-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{3}{2})-\Gamma(c-\frac{e}{2}+\frac{a}{2}+1)}+\{(a-1)(a-2)-c(e-2)\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+\frac{5}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-1)\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{1}{2})}}{\int_0^1 x^{c-1} (1-x)^{c-e+4} [1+\alpha x+\beta(1-x)]^{-2c+e-4} dx} \dots\dots\dots(4.2.6)$$

Provided $\text{Re}(c)>0, \text{Re}(c-e+4)>0, \text{Re}(e)>0$. Also the constants α and β are such that none of the expressions $1+\alpha, 1+\beta$ and $1+\alpha x+\beta(1-x), 0 \leq x \leq 1$, is not zero.

So by well known definition of probability distributions we have:

$$F(x)=\frac{\left[\frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3)\Gamma(c)\Gamma(a-2)}{2^{2a+2}(1+\alpha)^c(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+4)}\right]^x \left\{-(a-1)(a-2)+c(2c-e+2)\right\}\frac{\Gamma(c+2-\frac{e}{2}-\frac{a}{2})\Gamma(\frac{e}{2}-\frac{a}{2}+\frac{1}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-\frac{3}{2})-\Gamma(c-\frac{e}{2}+\frac{a}{2}+1)}+\{(a-1)(a-2)-c(e-2)\}\frac{\Gamma(c-\frac{e}{2}-\frac{a}{2}+\frac{5}{2})\Gamma(\frac{e}{2}-\frac{a}{2})}{\Gamma(\frac{e}{2}+\frac{a}{2}-1)\Gamma(c-\frac{e}{2}+\frac{a}{2}+\frac{1}{2})}}{\int_0^1 x^{c-1} (1-x)^{c-e+4} [1+\alpha x+\beta(1-x)]^{-2c+e-4} dx}$$

=0, elsewhere
=1, $\int_0^1 f(x)dx = 1$

Where $f(x) = F_1^2(a, 3 - a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Seventh Formula

$$\int_0^1 x^{c-1} (1-x)^{c-e+3} [1 + \alpha x + \beta(1-x)]^{-2c+e-4} F_1^2(a, 4-a, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx =$$

$$\frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3)\Gamma(c)\Gamma(a-3)}{2^{2a-3}(1+\alpha)^c(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+4)} x \{ \{e(2c-e)-(a-6)(a-c+e)-c-11\} \frac{\Gamma(c+2-\frac{e-a}{2})\Gamma(\frac{e-a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}-\frac{3}{2})\Gamma(c-\frac{e-a}{2}+\frac{a}{2})} + \{(-e(2c-e+a+2)+(a+3)(a+c+1)-6a) \frac{\Gamma(c-\frac{e-a}{2}+\frac{5}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2}-2)\Gamma(c-\frac{e-a}{2}+\frac{1}{2})}\} \dots\dots\dots(4.2.7)$$

Provided $\text{Re}(c)>0, \text{Re}(c-e+4)>0, \text{Re}(e)>0$. Also the constants α and β are such that none of the expressions $1+\alpha, 1+\beta$ and $1+\alpha x+\beta(1-x), 0 \leq x \leq 1$, is not zero.

So by well known definition of probability distributions we have:

$$F(x) = \frac{\left[\frac{\Gamma(e)\Gamma(c-e+4)\Gamma(e-c-3)\Gamma(c)\Gamma(a-3)}{2^{2a-3}(1+\alpha)^c(1+\beta)^{c-e+4}\Gamma(e-a)\Gamma(e-c)\Gamma(a)\Gamma(2c-e-a+4)} x \{ \{e(2c-e)-(a-6)(a-c+e)-c-11\} \frac{\Gamma(c+2-\frac{e-a}{2})\Gamma(\frac{e-a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}-\frac{3}{2})\Gamma(c-\frac{e-a}{2}+\frac{a}{2})} + \{(-e(2c-e+a+2)+(a+3)(a+c+1)-6a) \frac{\Gamma(c-\frac{e-a}{2}+\frac{5}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2}-2)\Gamma(c-\frac{e-a}{2}+\frac{1}{2})}\} \right]}{\int_0^1 x^{c-1} (1-x)^{c-e+3} [1 + \alpha x + \beta(1-x)]^{-2c+e-4} F_1^2(a, 4-a, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx}$$

$$F(x) = \int_0^1 x^{c-1} (1-x)^{c-e+3} [1 + \alpha x + \beta(1-x)]^{-2c+e-4} F_1^2(a, 4-a, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx$$

=0, elsewhere

$$=1, \int_0^1 f(x) dx = 1$$

Where $f(x) = F_1^2(a, 4 - a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Eighth Formula

$$\int_0^1 x^{c-1} (1-x)^{c-e} [1 + \alpha x + \beta(1-x)]^{-2c+e-1} F_1^2(a, 4-a, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx =$$

$$\frac{\Gamma(e)\Gamma(c-e+1)\Gamma(c-3)\Gamma(c)\Gamma(a-3)}{2^{2a-3}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a+1)} x \{ \{(a+3)(1+a-c)+e(2c-e+1)-6a\} \frac{\Gamma(c+\frac{1}{2}-\frac{e-a}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2}-2)\Gamma(c-\frac{e-a}{2}+\frac{3}{2})} + \{(-a-7)(a+c-2)-e(2c-e+1)-3(a-1)\} \frac{\Gamma(c+1-\frac{e-a}{2})\Gamma(\frac{e-a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}-\frac{3}{2})\Gamma(c-\frac{e-a}{2}+1)} \} \dots\dots\dots(4.2.8)$$

Provided $\text{Re}(c-3)>0, \text{Re}(c-e+1)>0, \text{Re}(e)>0$. Also the constants α and β are such that none of the expressions $1+\alpha, 1+\beta$ and $1+\alpha x+\beta(1-x), 0 \leq x \leq 1$, is not zero.

So by well known definition of probability distributions we have:

$$F(x) = \frac{\left[\frac{\Gamma(e)\Gamma(c-e+1)\Gamma(c-3)\Gamma(c)\Gamma(a-3)}{2^{2a-3}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a+1)} x \{ \{(a+3)(1+a-c)+e(2c-e+1)-6a\} \frac{\Gamma(c+\frac{1}{2}-\frac{e-a}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2}-2)\Gamma(c-\frac{e-a}{2}+\frac{3}{2})} + \{(-a-7)(a+c-2)-e(2c-e+1)-3(a-1)\} \frac{\Gamma(c+1-\frac{e-a}{2})\Gamma(\frac{e-a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}-\frac{3}{2})\Gamma(c-\frac{e-a}{2}+1)} \} \right]}{\int_0^1 x^{c-1} (1-x)^{c-e} [1 + \alpha x + \beta(1-x)]^{-2c+e-1} F_1^2(a, 4-a, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx}$$

=0, elsewhere

$$=1, \int_0^1 f(x) dx = 1$$

Where $f(x) = F_1^2(a, 4 - a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Ninth Formula

$$\int_0^1 x^{c-1} (1-x)^{c-e-1} [1+\alpha x+\beta(1-x)]^{-2c+e} F_1^2(a, 3-a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx =$$

$$\frac{\Gamma(e)\Gamma(c-e)\Gamma(c-3)\Gamma(a-2)}{2^{2a-2}(1+\alpha)^c(1+\beta)^{c-e}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a)} x \left[\{(a-1)(a-2)+(e-c)(2c-e-2)\} \frac{\Gamma(c-\frac{e-a}{2})\Gamma(\frac{e-a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}+\frac{3}{2})\Gamma(c-\frac{e-a}{2}-1)} + \{(a-1)(a-2)+(e-2)(c-e)\} \frac{\Gamma(c+\frac{1}{2}-\frac{e-a}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2}-1)\Gamma(c-\frac{e-a}{2}-\frac{3}{2})} \right] \dots (4.2.9)$$

Provided $\text{Re}(c-3)>0, \text{Re}(c-e)>0, \text{Re}(e)>0$. Also the constants α and β are such that none of the expressions $1+\alpha, 1+\beta$ and $1+\alpha x+\beta(1-x), 0 \leq x \leq 1$, is not zero.

So by well known definition of probability distributions we have:

$$F(x) = \frac{\left[\frac{\Gamma(e)\Gamma(c-e)\Gamma(c-3)\Gamma(a-2)}{2^{2a-2}(1+\alpha)^c(1+\beta)^{c-e}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a)} x \left\{ \{(a-1)(a-2)+(e-c)(2c-e-2)\} \frac{\Gamma(c-\frac{e-a}{2})\Gamma(\frac{e-a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}+\frac{3}{2})\Gamma(c-\frac{e-a}{2}-1)} + \{(a-1)(a-2)+(e-2)(c-e)\} \frac{\Gamma(c+\frac{1}{2}-\frac{e-a}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2}-1)\Gamma(c-\frac{e-a}{2}-\frac{3}{2})} \right\} \right]}{\int_0^1 x^{c-1} (1-x)^{c-e-1} [1+\alpha x+\beta(1-x)]^{-2c+e} dx}$$

=0, elsewhere

$$=1, \int_0^1 f(x) dx = 1$$

Where $f(x) = F_1^2(a, 3 - a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Tenth Formula

$$\int_0^1 x^{c-1} (1-x)^{c-e-2} [1+\alpha x+\beta(1-x)]^{-2c+e+1} F_1^2(a, 2-a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx =$$

$$\frac{\Gamma(e)\Gamma(c-e-1)\Gamma(c-3)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e-1}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a-1)} x \left[\{-(a-1)(a+1)+c(a+c-2)-e(2c-e-1)\} \frac{\Gamma(c-\frac{1}{2}-\frac{e-a}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2}+1)\Gamma(c-\frac{1}{2}+\frac{e-a}{2})} + \{(a-1)(a-3)-c(c-a)+e(2c-e-1)\} \frac{\Gamma(c-\frac{e-a}{2})\Gamma(\frac{e-a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}+\frac{1}{2})\Gamma(c-\frac{e-a}{2}-1)} \right] \dots (4.2.10)$$

Provided $\text{Re}(c-3)>0, \text{Re}(c-e-1)>0, \text{Re}(e)>0$. Also the constants α and β are such that none of the expressions $1+\alpha, 1+\beta$ and $1+\alpha x+\beta(1-x), 0 \leq x \leq 1$, is not zero.

So by well known definition of probability distributions we have:

$$F(x) = \frac{\left[\frac{\Gamma(e)\Gamma(c-e-1)\Gamma(c-3)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e-1}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a-1)} x \left\{ \{-(a-1)(a+1)+c(a+c-2)-e(2c-e-1)\} \frac{\Gamma(c-\frac{1}{2}-\frac{e-a}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2}+1)\Gamma(c-\frac{1}{2}+\frac{e-a}{2})} + \{(a-1)(a-3)-c(c-a)+e(2c-e-1)\} \frac{\Gamma(c-\frac{e-a}{2})\Gamma(\frac{e-a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}+\frac{1}{2})\Gamma(c-\frac{e-a}{2}-1)} \right\} \right]}{\int_0^1 x^{c-1} (1-x)^{c-e-2} [1+\alpha x+\beta(1-x)]^{-2c+e+1} dx}$$

=0, elsewhere

$$=1, \int_0^1 f(x) dx = 1$$

$$\text{Where } f(x) = F_1^2(a, 2-a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$$

Eleventh Formula

$$\int_0^1 x^{c-1} (1-x)^{c-e-3} [1+\alpha x+\beta(1-x)]^{-2c+e+2} F_1^2(a, 1-a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx =$$

$$\frac{\Gamma(e)\Gamma(c-e-2)\Gamma(c-3)}{2^{2a}(1+\alpha)^c(1+\beta)^{c-e-2}\Gamma(e-a)\Gamma(2c-e-a-2)} x \{ \{-(a+2)(a-3)+3c(c-3)-e(3c-e-4)\} \frac{\Gamma(c-1-\frac{e-a}{2})\Gamma(\frac{e-a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}+\frac{1}{2})\Gamma(c-\frac{1}{2}e+\frac{a}{2}-1)} + \{-(a+1)(a-2)+c(c-3)+e(e-\frac{1}{2}-\frac{e-a}{2})\} \frac{\Gamma(c-\frac{1}{2}-\frac{e-a}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-\frac{3}{2})} \} \dots \dots \dots (4.2.11)$$

Provided $\text{Re}(c-3)>0, \text{Re}(c-e-2)>0, \text{Re}(e)>0$. Also the constants α and β are such that none of the expressions $1+\alpha, 1+\beta$ and $1+\alpha x+\beta(1-x), 0 \leq x \leq 1$, is not zero.

So by well known definition of probability distributions we have:

$$F(x) = \frac{\left[\frac{\Gamma(e)\Gamma(c-e-2)\Gamma(c-3)}{2^{2a}(1+\alpha)^c(1+\beta)^{c-e-2}\Gamma(e-a)\Gamma(2c-e-a-2)} x \{ \{-(a+2)(a-3)+3c(c-3)-e(3c-e-4)\} \frac{\Gamma(c-1-\frac{e-a}{2})\Gamma(\frac{e-a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}+\frac{1}{2})\Gamma(c-\frac{1}{2}e+\frac{a}{2}-1)} + \{-(a+1)(a-2)+c(c-3)+e(e-\frac{1}{2}-\frac{e-a}{2})\} \frac{\Gamma(c-\frac{1}{2}-\frac{e-a}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-\frac{3}{2})} \} \right]}{\int_0^1 x^{c-1} (1-x)^{c-e-3} [1+\alpha x+\beta(1-x)]^{-2c+e+2} dx}$$

=0, elsewhere

$$=1, \int_0^1 f(x) dx = 1$$

$$\text{Where } f(x) = F_1^2(a, 1-a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$$

Twelfth Formula

$$\int_0^1 x^{c-1} (1-x)^{c-e-3} [1+\alpha x+\beta(1-x)]^{-2c+e+2} F_1^2(a, -a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx =$$

$$\frac{\Gamma(e)\Gamma(c-e-2)\Gamma(c-2)}{2^{2a+1}(1+\alpha)^c(1+\beta)^{c-e-2}\Gamma(e-a)\Gamma(2c-e-a-2)} x \{ \{-(a+1)(a+2)+a(c-e)+c(c-3)\} \frac{\Gamma(c-1-\frac{e-a}{2})\Gamma(\frac{e-a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}+\frac{1}{2})\Gamma(c-\frac{1}{2}e+\frac{a}{2}-1)} + \{-(a+1)(a-2)-a(c-e)+c(c-3)\} \frac{\Gamma(c-\frac{1}{2}-\frac{e-a}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-\frac{1}{2})} \} \dots \dots \dots (4.2.12)$$

Provided $\text{Re}(c)>2, \text{Re}(c-e-2)>0, \text{Re}(e)>0$. Also the constants α and β are such that none of the expressions $1+\alpha, 1+\beta$ and $1+\alpha x+\beta(1-x), 0 \leq x \leq 1$, is not zero.

So by well known definition of probability distributions we have:

$$F(x) = \frac{\left[\frac{\Gamma(e)\Gamma(c-e-2)\Gamma(c-2)}{2^{2a+1}(1+\alpha)^c(1+\beta)^{c-e-2}\Gamma(e-a)\Gamma(2c-e-a-2)} x \{ \{-(a+1)(a+2)+a(c-e)+c(c-3)\} \frac{\Gamma(c-1-\frac{e-a}{2})\Gamma(\frac{e-a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}+\frac{1}{2})\Gamma(c-\frac{1}{2}e+\frac{a}{2}-1)} + \{-(a+1)(a-2)-a(c-e)+c(c-3)\} \frac{\Gamma(c-\frac{1}{2}-\frac{e-a}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2})\Gamma(c-\frac{e}{2}+\frac{a}{2}-\frac{1}{2})} \} \right]}{\int_0^1 x^{c-1} (1-x)^{c-e-3} [1+\alpha x+\beta(1-x)]^{-2c+e+2} dx}$$

=0, elsewhere

$$=1, \int_0^1 f(x) dx = 1$$

Where $f(x) = F_1^2(a, -a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

Thirteen Formula

$$\int_0^1 x^{c-1} (1-x)^{c-e-3} [1+\alpha x+\beta(1-x)]^{-2c+e+2} F_1^2(a, -1; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}) dx =$$

$$\frac{\Gamma(e)\Gamma(c-e-2)\Gamma(c-1)}{2^{2a+2}(1+\alpha)^c(1+\beta)^{c-e-2}\Gamma(e-a)\Gamma(2c-e-a-2)} x \{ \{-(a-1)(a+2)+(c-1)(2c-e)-$$

$$2c\} \frac{\Gamma(c-1-\frac{e-a}{2}-\frac{1}{2})\Gamma(\frac{e-a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}+\frac{1}{2})\Gamma(c-\frac{e-a}{2})} + \{ -a(a+1)+e(c-$$

$$1) \} \frac{\Gamma(c-\frac{1}{2}-\frac{e-a}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2}+1)\Gamma(c-\frac{e-a}{2}-\frac{1}{2})}] \dots \dots \dots (4.2.13)$$

Provided $\text{Re}(c)>1, \text{Re}(c-e-2)>0, \text{Re}(e)>0$. Also the constants α and β are such that none of the expressions $1+\alpha, 1+\beta$ and $1+\alpha x+\beta(1-x), 0 \leq x \leq 1$, is not zero.

So by well known definition of probability distributions we have:

$$F(x) = \frac{\left[\frac{\Gamma(e)\Gamma(c-e-2)\Gamma(c-1)}{2^{2a+1}(1+\alpha)^c(1+\beta)^{c-e-2}\Gamma(e-a)\Gamma(2c-e-a-2)} x \right. \\ \left. \{ \{-(a-1)(a+2)+(c-1)(2c-e)-2c\} \frac{\Gamma(c-1-\frac{e-a}{2}-\frac{1}{2})\Gamma(\frac{e-a}{2}+\frac{1}{2})}{\Gamma(\frac{e-a}{2}+\frac{1}{2})\Gamma(c-\frac{e-a}{2})} + \right. \\ \left. + \{ -a(a+1)+e(c-1) \} \frac{\Gamma(c-\frac{1}{2}-\frac{e-a}{2})\Gamma(\frac{e-a}{2})}{\Gamma(\frac{e-a}{2}+1)\Gamma(c-\frac{e-a}{2}-\frac{1}{2})} \right]}{\int_0^1 x^{c-1} (1-x)^{c-e-3} [1+\alpha x+\beta(1-x)]^{-2c+e+2} dx}$$

=0, elsewhere

$$=1, \int_0^1 f(x) dx = 1$$

Where $f(x) = F_1^2(a, -1 - a; e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)})$

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