Integral Involving Hypergeometric Function, Ĥ and Ī Function With Probability Distribution

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5.1 INTRODUCTION

This chapter we finalize the probability distribution of seven integrals hypergeometric function and Ī-function.

The results are obtained with the help of seven integrals involving hypergeometric function.

Since Ī-function is one of the most generalized function of one variable studied so far, it not only contains Meijer's G-Function, Fox's H-function and Inayat Hussain's H-function as special cases, but also inludes most of the commonly used functions. Therefore from our results, a large number of known as well as unknown results for G, H and Ĥ-function can be obtained.

5.2 RESULTS REQUIRED

The following seven integrals involving hypergeometric functions obtained earlier by Nagar [108] will be required in our present investigations then finalize probability distributions.

First Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1}F_1^2\left(a,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx\,\Gamma =$$

$$\frac{\Gamma(\mathrm{e})\Gamma(\mathrm{c-e+1})\Gamma(\mathrm{c})}{2^{2a}(1+\alpha)^e(1+\beta)^{c-e+1}\Gamma(\mathrm{e-1})\Gamma 2\mathrm{c-e-a+1})}X\frac{[\Gamma\left(\mathrm{c}-\frac{1}{2\mathrm{e}}-\frac{1}{2\mathrm{a}}+\frac{1}{2}\right)\Gamma\left(\frac{1}{2\mathrm{e}}-\frac{1}{2\mathrm{a}}\right)]}{[\Gamma\left(\frac{1}{2\mathrm{e}}+\frac{1}{2\mathrm{a}}\right)\Gamma\left(\mathrm{c}-\frac{1}{2\mathrm{e}}+\frac{1}{2\mathrm{a}}+\frac{1}{2}\right)]}\ldots\ldots\ldots\ldots\ldots(5.2.1)$$

Provided Re(c) >0, Re (e) >0 and Re(c-e+1) >0. Also the constants α and β are such that none of the expression 1+α, 1+β, 1+αx+β(1-x), where 0≤x≤1, is not zero.

Then by definition of probability distribution, we have from (5.2.1)

$$f(x)=\frac{\frac{\Gamma(\mathrm{e})\Gamma(\mathrm{c-e+1})\Gamma(\mathrm{c})}{2^{2a}(+\alpha)^e(1+\beta)^{c-e+1}\Gamma(\mathrm{e-1})\Gamma 2\mathrm{c-e-a+1})}X\frac{[\Gamma\left(\mathrm{c}-\frac{1}{2\mathrm{e}}-\frac{1}{2\mathrm{a}}+\frac{1}{2}\right)\Gamma\left(\frac{1}{2\mathrm{e}}-\frac{1}{2\mathrm{a}}\right)]}{[\Gamma\left(\frac{1}{2\mathrm{e}}+\frac{1}{2\mathrm{a}}\right)\Gamma\left(\mathrm{c}-\frac{1}{2\mathrm{e}}+\frac{1}{2\mathrm{a}}+\frac{1}{2}\right)]}}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1}F_1^2\left(a,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx}=0$$

Elsewhere, $\int_0^1 f(x)dx=1$

Where f(x) = $F_1^2\left(\alpha,1-\alpha,e;\ \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$

Second Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1}F_1^2\left(a,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx\,\Gamma =$$

$$\frac{\Gamma(\mathrm{e})\Gamma(\mathrm{c-e+1})\Gamma(\mathrm{c-1})\Gamma(\mathrm{a-1})}{2^{2a-1}(1+\alpha)^e(1+\beta)^{c-e+1}\Gamma(\mathrm{e-1})\Gamma(\mathrm{a})\Gamma 2\mathrm{c-e-a+1})}$$

$$X\frac{[\Gamma\left(\mathrm{c}-\frac{1}{2\mathrm{e}}-\frac{1}{2\mathrm{a}}+\frac{1}{2}\right)\Gamma\left(\frac{1}{2\mathrm{e}}-\frac{1}{2\mathrm{a}}\right)]}{[\Gamma\left(\frac{1}{2\mathrm{e}}+\frac{1}{2\mathrm{a}}\right)\Gamma\left(\mathrm{c}-\frac{1}{2\mathrm{e}}+\frac{1}{2\mathrm{a}}+\frac{1}{2}\right)]}\ldots\ldots\ldots(5.2.2)$$

Provided Re(c) >0, Re (e) >0 and Re(c-e+1) >0. Also the constants α and β are such that none of the expression 1+α, 1+β, 1+αx+β(1-x), where 0≤x≤1, is not zero.

Then by definition of probability distribution, we have from (5.2.2)

$$f(x)=\frac{\frac{\Gamma(\mathrm{e})\Gamma(\mathrm{c-e+1})\Gamma(\mathrm{c-1})\Gamma(\mathrm{a-1})}{2^{2a-1}(1+\alpha)^e(1+\beta)^{c-e+1}\Gamma(\mathrm{e-1})\Gamma(\mathrm{a})\Gamma 2\mathrm{c-e-a+1})}X\frac{[\Gamma\left(\mathrm{c}-\frac{1}{2\mathrm{e}}-\frac{1}{2\mathrm{a}}+\frac{1}{2}\right)\Gamma\left(\frac{1}{2\mathrm{e}}-\frac{1}{2\mathrm{a}}\right)]}{[\Gamma\left(\frac{1}{2\mathrm{e}}+\frac{1}{2\mathrm{a}}\right)\Gamma\left(\mathrm{c}-\frac{1}{2\mathrm{e}}+\frac{1}{2\mathrm{a}}+\frac{1}{2}\right)]}}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1}F_1^2\left(a,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx}$$

$$=0$$

Elsewhere, $\int_0^1 f(x)dx=1$

Where f(x) = $F_1^2\left(\alpha,1-\alpha,e;\ \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$

Third Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1}F_1^2\left(\alpha,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx = \frac{\Gamma(e)\Gamma(c-e+1)\Gamma(c)}{2^{2a+1}(1+\alpha)^e(1+\beta)^{c-e+1}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a+1)} X \frac{[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)]}{[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}\right)]} - \frac{[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+1\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)]}{[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+1\right)]} \quad \text{.........(5.2.3)}$$

Provided Re(c) >0, Re (e) >0 and Re(c-e+1) >0. Also the constants α and β are such that none of the expression 1+α, 1+β, 1+αx+β(1-x), where 0≤x≤1, is not zero.

Then by definition of probability distribution, we have from (5.2.2)

$$f(x) = \frac{\frac{\Gamma(e)\Gamma(c-e+1)\Gamma(c)}{2^{2a+1}(1+\alpha)^e(1+\beta)^{c-e+1}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a+1)} X \frac{[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)]}{[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}\right)]} - \frac{[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+1\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)]}{[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+1\right)]}}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1} F_1^2\left(\alpha,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx} = 0$$

Elsewhere, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2\left(\alpha,1-\alpha,e;\ \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$

Fourth Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-2}F_1^2\left(\alpha,2-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx = \frac{\Gamma(e)\Gamma(c-e+2)\Gamma(c)\Gamma(a-1)\Gamma(e-c-1)}{2^{2a-1}(1+\alpha)^e(1+\beta)^{c-e+2}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a+2)} X \frac{[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)]}{[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}\right)]} - \frac{[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+1\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)]}{[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+1\right)]} \quad \text{.........(5.2.4)}$$

Provided Re(c) >0, Re (e) >0 and Re(c-e+1) >0. Also the constants α and β are such that none of the expression 1+α, 1+β, 1+αx+β(1-x), where 0≤x≤1, is not zero.

Then by definition of probability distribution, we have from (5.2.2)

$$f(x) = \frac{\frac{\Gamma(e)\Gamma(c-e+2)\Gamma(c)\Gamma(a-1)\Gamma(e-c-1)}{2^{2a-1}(1+\alpha)^e(1+\beta)^{c-e+2}\Gamma(e-a)\Gamma(a)\Gamma(2c-e-a+2)} X \frac{[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)]}{[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}\right)]} - \frac{[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+1\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)]}{[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+1\right)]}}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1} F_1^2\left(\alpha,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx}$$

Elsewhere, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2\left(\alpha,1-\alpha,e;\ \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$

Fifth Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e+1}[.1+\alpha x+\beta(1-x)]^{-2c+e-2}F_1^2\left(\alpha,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx =$$

$$\frac{\Gamma(e)\Gamma(c-e+2)\Gamma(c)\Gamma(e-c+1)}{2^{2a}(1+\alpha)^e(1+\beta)^{c-e+2}\Gamma(e-a)\Gamma(e-c)\Gamma(2c-e-a+2)}X\frac{\left[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+1\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}-\frac{1}{2}\right)\right]}{\left[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}\right)\right]}-$$

$$\frac{\left[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+\frac{3}{2}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}-\frac{1}{2a}\right)\right]}{\left[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}\right)\right]}\text{.........(5.2.5)}$$

Provided Re(c) >0, Re (e) >0 and Re(c-e+1) >0. Also the constants α and β are such that none of the expression 1+α, 1+β, 1+αx+β(1-x), where 0≤x≤1, is not zero.
Then by definition of probability distribution, we have from (5.2.2)

$$f(x)=\frac{\dfrac{\Gamma(e)\Gamma(c-e+2)\Gamma(c)\Gamma(a-1)\Gamma(e-c+1)}{2^{2a}(1+\alpha)^e(1+\beta)^{c-e+2}\Gamma(e-a)\Gamma(e-c)\Gamma(2c-e-a+2)}X\dfrac{\left[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+1\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)\right]}{\left[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+1\right)\right]}-\dfrac{\left[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+\frac{3}{2}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}-\frac{1}{2}\right)\right]}{\left[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}\right)\right]}}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1}F_1^2\left(\alpha,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx}=0$$

Elsewhere, $\int_0^1 f(x)dx=1$

Where f(x) = $F_1^2\left(\alpha,1-\alpha,e;\ \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$

Sixth Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e+1}[.1+\alpha x+\beta(1-x)]^{-2c+e-2}F_1^2\left(\alpha,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx =$$

$$\frac{\Gamma(e)\Gamma(c-e)\Gamma(c)\Gamma(c)}{2^{2a+1}(1+\alpha)^e(1+\beta)^{c-e}\Gamma(e-a)\Gamma(e-c)\Gamma(2c-e-a)}X\frac{\left[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)\right]}{\left[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2a}\right)\right]}-$$

$$\frac{\left[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)\right]}{\left[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}\right)\right]}\text{.........(5.2.6)}$$

Provided Re(c) >0, Re (e) >0 and Re(c-e+1) >0. Also the constants α and β are such that none of the expression 1+α, 1+β, 1+αx+β(1-x), where 0≤x≤1, is not zero.
Then by definition of probability distribution, we have from (5.2.2)

$$f(x)=\frac{\dfrac{\Gamma(e)\Gamma(c-e)\Gamma(c)}{2^{2a+1}(1+\alpha)^e(1+\beta)^{c-e}\Gamma(e-a)\Gamma(2c-e-a)}X\dfrac{\left[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)\right]}{\left[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2a}\right)\right]}+\dfrac{\left[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)\right]}{\left[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+\frac{1}{2}\right)\right]}}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}=0$$

Elsewhere, $\int_0^1 f(x)dx=1$

Where f(x) = $F_1^2\left(\alpha,1-\alpha,e;\ \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$

Seventh Formula

$$\int_0^1 x^{c-1}(1-x)^{c-e-1}[.1+\alpha x+\beta(1-x)]^{-2c+e}F_1^2\left(a,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)dx =$$

$$\frac{\Gamma(e)\Gamma(c-e)\Gamma(c)\Gamma(c-1)}{2^{2a}(1+\alpha)^e(1+\beta)^{c-e}\Gamma(e-a)\Gamma(e-c)\Gamma(2c-e-a)}X\frac{[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)]}{[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+1\right)]}+$$

$$\frac{[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)]}{[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)]}\ldots\ldots\ldots(5.2.7)$$

Provided Re(c) >0, Re (e) >0 and Re(c-e+1) >0. Also the constants α and β are such that none of the expression 1+α, 1+β, 1+αx+β(1-x), where 0≤x≤1, is not zero.
Then by definition of probability distribution, we have from (5.2.2)

$$f(x) = \frac{\frac{\Gamma(e)\Gamma(c-e)\Gamma(c-1)}{2^{2a+1}(1+\alpha)^e(1+\beta)^{c-e}\Gamma(e-a)\Gamma(2c-e-a)}X\frac{[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)]}{[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}+1\right)]}+\frac{[\Gamma\left(c-\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)]}{[\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma\left(c-\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)]}}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

$= 0$

Elsewhere, $\int_0^1 f(x)dx = 1$

Where f(x) = $F_1^2\left(\alpha,1-\alpha,e;\ \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$

MAIN INTEGRALS

In this section, the following probability distribution of seven integrals involving hypergeometric function and Ī-function will be evaluated.

First Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}F_1^2\left(a,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{I}_{p,q}^{m,n}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\middle|\begin{matrix}{}_j(\alpha_j,A_j;a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\end{matrix}\right\}dx =\right.$$

$$=\frac{\Gamma(e)\Gamma(\frac{1}{2e}-\frac{1}{2a})}{2^{2a}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma(e-a)}*$$

$$\bar{I}_{p+3,q+2}^{m,n+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\begin{matrix}(e-c,\lambda;1),.(1-c,\lambda;1).(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1)_{,i}(\alpha_j,A_j;\ a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\ (e+a-2c,2\lambda;1)(\frac{1}{2}+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1)\end{matrix}\right\}\right.\ldots\ldots\ldots\ldots(5.3.1)$$

Provided ʎ>0, Re(c-e+1+ʎ) $\min_{1\le j\le m}(\beta_j/B_j) > 0$, Re(c+ʎ) $\min_{1\le j\le m}(\beta_j/B_j) > 0$ and Re(c-1/2e-1/2a+1/2+ʎ) $\min_{1\le j\le m}(\beta_j/B_j) > 0$, ø>0, $|arg(z)| < \theta_{\pi/2}$, where θ is same as given. Also the constants α and β such that none of the expressions 1+α, 1+β, [1+αx+β(1-x)], where 0≤x≤1, is not zero.

Then by definition of probability distribution, we have from (5.3.1):

$$f(x) = \frac{\frac{\Gamma(e)\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{2^{2a}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma(e-a)}*\bar{I}_{p+3,q+2}^{m,n+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\begin{matrix}(e-c,\lambda;1),.(1-c,\lambda;1).\\(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1)_{,i}(\alpha_j,A_j;\ a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\\(e+a-2c,2\lambda;1)(\frac{1}{2}+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1)\end{matrix}\right\}\right.}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx = 1$

Where

$$f(x) = F_1^2\left(a, 1-\alpha, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right) \bar{I}_{p,q}^{m,n}\left(\left\{ z \frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}} \middle| \frac{\square_j(\alpha_j,A_j;a_j)_p}{\square_j(\beta_j,B_j;1)_{m.m+1}\ \square_j(\beta,B_j;b_j)_q} \right\}\right)$$

Second Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1} F_1^2\left(a, 2-\alpha, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right) \bar{I}_{p,q}^{m,n}\left(\left\{ z \frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}} \middle| \frac{\square_j(\alpha_j,A_j;a_j)_p}{\square_j(\beta_j,B_j;1)_{m.m+1}\ \square_j(\beta,B_j;b_j)_q} \right\}\right) dx =$$

$$= \frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(e-a)} * U$$

$$\bar{I}_{p+3,q+2}^{m,n+1}\left(\left\{ \frac{z}{(1+\alpha)(1+\beta)} \middle| \begin{matrix} (e-c,\lambda;1),.(2-c,\lambda;1) \\ .\left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;\,a_j)_p \\ \hline \square_j(\beta_j,B_j;1)_{m.m+1}\ \square_j(\beta,B_j;b_j)_q \\ (e+a-2c,2\lambda;1)\left(\frac{3}{2}+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right) \end{matrix} \right\}\right) -$$

$$V.\,\bar{I}_{p+3,q+2}^{m,n+1}\left(\left\{ \frac{z}{(1+\alpha)(1+\beta)} \middle| \begin{matrix} (e-c,\lambda;1),.(2-c,\lambda;1). \\ \left(\frac{1}{2e}+\frac{1}{2a}-c,\lambda;1\right)_{,i}(\alpha_j,A_j;\,a_j)_p \\ \hline \square_j(\beta_j,B_j;1)_{m.m+1}\ \square_j(\beta,B_j;b_j)_q \\ (e+a-2c,2\lambda;1)\left(1-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right) \end{matrix} \right\}\right) \ldots\ldots(5.3.2)$$

Provided ʎ>0, Re(c-e+1+ʎ) $\min_{1\le j\le m}(\beta_j/B_j) > 0$, Re(c+ʎ) $\min_{1\le j\le m}(\beta_j/B_j) > 0$ and Re(c-1/2e-1/2a+1/2+ʎ) $\min_{1\le j\le m}(\beta_j/B_j) > 0$, ø>0, $|arg(z)| < \theta_{\pi/2}$, where θ is same as given. Also the constants α and β such that none of the expressions 1+α, 1+β, [1+αx+β(1-x)], where 0≤x≤1, is not zero.

$$U = \frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)},\quad V = \frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)}$$

Then by definition of probability distribution, we have from (5.3.2): $(x) =$

$$\frac{\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(e-a)}\left(*U\bar{I}_{p+3,q+2}^{m,n+1}\left(\left\{ \frac{z}{(1+\alpha)(1+\beta)} \middle| \begin{matrix} (e-c,\lambda;1),.(2-c,\lambda;1). \\ \left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;\,a_j)_p \\ \hline \square_j(\beta_j,B_j;1)_{m.m+1}\ \square_j(\beta,B_j;b_j)_q \\ (e+a-2c,2\lambda;1)\left(\frac{3}{2}+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right) \end{matrix} \right\}\right) - V.\bar{I}_{p+3,q+2}^{m,n+1}\left(\left\{ \frac{z}{(1+\alpha)(1+\beta)} \middle| \begin{matrix} (e-c,\lambda;1),.(2-c,\lambda;1). \\ \left(\frac{1}{2e}+\frac{1}{2a}-c,\lambda;1\right)_{,i}(\alpha_j,A_j;\,a_j)_p \\ \hline \square_j(\beta_j,B_j;1)_{m.m+1}\ \square_j(\beta,B_j;b_j)_q \\ (e+a-2c,2\lambda;1)\left(1-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right) \end{matrix} \right\}\right)\right)}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

$=0$

Elsewhere, $\int_0^1 f(x)dx = 1$

Where

$$f(x) = F_1^2\left(a, 2-\alpha, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right) \bar{I}_{p,q}^{m,n}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\middle| \frac{{}_j(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q}\right\}\right)$$

Third Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1}F_1^2\left(a,-\alpha,e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{I}_{p,q}^{m,n}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\middle| \frac{{}_j(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q}\right\}dx\right.$$

$$=\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(e-a)}*U$$

$$\bar{I}_{p+3,q+2}^{m,n+1}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle| \frac{(e-c,\lambda;1),.(2-c,\lambda;1).\left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\ (e+a-2c,2\lambda;1)\left(\frac{1}{2}+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)}\right\}\right)-$$

$$V.\ \bar{I}_{p+3,q+2}^{m,n+1}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle| \frac{(e-c,\lambda;1),.(2-c,\lambda;1).\left(\frac{1}{2e}+\frac{1}{2a}-c,\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\ (e+a-2c,2\lambda;1)\left(\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)}\right\}\right)\ldots$$

…(5.3.3)

Provided λ>0, Re(c-e+1+λ) $\min_{1\le j\le m}(\beta_j/B_j) > 0$, Re(c+$\lambda$) $\min_{1\le j\le m}(\beta_j/B_j) > 0$ and Re(c-1/2e-1/2a+1/2+λ) $\min_{1\le j\le m}(\beta_j/B_j) > 0$, ø>0, $|arg(z)| < \theta_{\pi/2}$, where θ is same as given. Also the constants α and β such that none of the expressions 1+α, 1+β, [1+αx+β(1-x)], where 0≤x≤1, is not zero.

$$U=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)},\ V=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)}$$

Then by definition of probability distribution, we have from (5.3.3):

$$f(x) = \frac{\dfrac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(e-a)}*\ U\bar{I}_{p+3,q+2}^{m,n+1}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle| \frac{(e-c,\lambda;1),.(2-c,\lambda;1).\left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\ (e+a-2c,2\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right)}\right\}\right) - V.\bar{I}_{p+3,q+2}^{m,n+1}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle| \frac{(e-c,\lambda;1),.(2-c,\lambda;1).\left(\frac{1}{2e}+\frac{1}{2a}-c,\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\ (e+a-2c,2\lambda;1)\left(\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)}\right\}\right)}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx = 1$

Where

$$f(x) = F_1^2\left(a, 2-\alpha, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right) \bar{I}_{p,q}^{m,n}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\middle| \frac{{}_j(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q}\right\}\right)$$

Fourth Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1} F_1^2\left(a,-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right) \bar{I}_{p,q}^{m,n}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}{}_j(\alpha_j,A_j;a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\end{matrix}\right.\right\}dx\right.$$

$$=\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(e-a)}*U$$

$$\bar{I}_{p+2,q+3}^{m+1,n+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda;1),.(1-c,\lambda;1).\\ \left(\frac{1}{2e}+\frac{1}{2a},-c\,\lambda;1\right)_i(\alpha_j,A_j;\,a_j)_p(e-c,\lambda;1)\\ \hline (e-c-1,\lambda;1)\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta_j,B_j;b_j)_q\\ (e+a-2c-1,,2\lambda;1)\left(1-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right)\end{matrix}\right.\right\}\right)-$$

$$V.\,\bar{I}_{p+2,q+3}^{m+1,n+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c,\lambda;1),.(1-c,\lambda;1).\\ \left(\frac{1}{2e}+\frac{1}{2a}-c-1,\lambda;1\right)_i(\alpha_j,A_j;\,a_j)_p(e-c,\lambda;1)\\ \hline (e-c,\lambda;1)\ {}_j(\beta_j,B_j;1)_{m.m-1}\ {}_j(\beta_j,B_j;b_j)_q\\ (e+a-2c,1\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right)\end{matrix}\right.\right\}\right)(5.3.4)$$

Provided ʎ>0, Re(c-e+1+ʎ) $\min_{1\le j\le m}(\beta_j/B_j)>0$, Re(c+ʎ) $\min_{1\le j\le m}(\beta_j/B_j)>0$ and Re(c-1/2e-1/2a+1/2+ʎ) $\min_{1\le j\le m}(\beta_j/B_j)>0$, ø>0, $|arg(z)|<\theta_{\pi/2}$, where θ is same as given. Also the constants α and β such that none of the expressions 1+α, 1+β, [1+αx+β(1-x)], where 0≤x≤1, is not zero.

$$U=\frac{\Gamma(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2})}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)},V=\frac{\Gamma(\frac{1}{2e}-\frac{1}{2a})}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)}$$

Then by definition of probability distribution, we have from (5.3.4):

$$f(x)=\frac{\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(e-a)} * U\bar{I}_{p+2,q+3}^{m+1,n+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda;1),.(1-c,\lambda;1).\\ \left(\frac{1}{2e}+\frac{1}{2a},-c\,\lambda;1\right)_i(\alpha_j,A_j;\,a_j)_p(e-c,\,\lambda;1)\\ \hline (e-c-1,\lambda;1)\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta_j,B_j;b_j)_q\\ (e+a-2c-1,,2\lambda;1)\left(1-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right)\end{matrix}\right.\right\}\right) - V.\bar{I}_{p+2,q+3}^{m+1,n+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c,\lambda;1),.(1-c,\lambda;1)\\ .\left(\frac{1}{2e}+\frac{1}{2a}-c-1,\lambda;1\right)_i(\alpha_j,A_j;\,a_j)_p(e-c,\lambda;1)\\ \hline (e-c,\lambda;1)\ {}_j(\beta_j,B_j;1)_{m.m-1}\ {}_j(\beta_j,B_j;b_j)_q\\ (e+a-2c,1\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right)\end{matrix}\right.\right\}\right.}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx=1$

Where

$$f(x)=F_1^2\left(a,2-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{I}_{p,q}^{m,n}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}{}_j(\alpha_j,A_j;a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\end{matrix}\right.\right\}\right.$$

Fifth Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e+1}[.1+\alpha x+\beta(1-x)]^{-2c+e-2} F_1^2\left(\alpha, 1-\alpha, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right) \bar{I}_{p,q}^{m,n}\left(\left\{z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\middle| \begin{matrix} {}_j(\alpha_j,A_j;a_j)_p \\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q \end{matrix}\right\}dx\right.$$

$$=\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(e-a)}*U$$

$$\bar{I}_{p+4,,q+3}^{m+1,n+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle| \begin{matrix} (e-c-1,\lambda;1),.(1-c,\lambda;1). \\ \left(\frac{1}{2e}+\frac{1}{2a'}-c\ \lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p(e-c,\lambda;1) \\ \hline (e-c-1,\lambda;1)\ {}_j(\beta_j,B_j;1)_{m.m+1} \\ {}_j(\beta_j,B_j;b_j)_q(e+a-2c-1,,2\lambda;1)\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}-c,\lambda;1\right) \end{matrix}\right\}\right)\text{-}$$

$$\text{v. } \bar{I}_{p+2,q+3}^{m+1,n+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle| \begin{matrix} (e-c-1,\lambda;1),.(1-c,\lambda;1). \\ \left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}-c1,\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p(e-c,\lambda;1) \\ \hline (e-c+1,\lambda;1)\ {}_j(\beta_j,B_j;1)_{m.m-1}\ {}_j(\beta_j,B_j;b_j)_q \\ (e+a-2c-1,2\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right) \end{matrix}\right\}\right)$$

……………………………………………………………………………………..(5.3.5)

Provided ʎ>0, Re(c-e+1+ʎ) $\min_{1\le j\le m}(\beta_j/B_j)>0$, Re(c+ʎ) $\min_{1\le j\le m}(\beta_j/B_j)>0$ and Re(c-1/2e-1/2a+1/2+ʎ) $\min_{1\le j\le m}(\beta_j/B_j)>0$, ø>0, $|arg(z)|<\theta_{\pi/2}$, where θ is same as given. Also the constants α and β such that none of the expressions 1+α, 1+β, [1+αx+β(1-x)], where 0≤x≤1, is not zero.

$$U=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)}, V=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)}$$

Then by definition of probability distribution, we have from (5.3.5): $f(x)=$

$$\frac{\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(e-a)} \left(*U\bar{I}_{p+4,,q+3}^{m+1,n+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle| \begin{matrix} (e-c-1,\lambda;1),.(1-c,\lambda;1). \\ \left(\frac{1}{2e}+\frac{1}{2a'}-c\ \lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p(e-c,\lambda;1) \\ \hline (e-c-1,\lambda;1)\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta_j,B_j;b_j)_q \\ (e+a-2c-1,,2\lambda;1)\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}-c,\lambda;1\right) \end{matrix}\right\}\right) -V.\bar{I}_{p+2,q+3}^{m+1,n+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle| \begin{matrix} (e-c-1,\lambda;1),.(1-c,\lambda;1). \\ \left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}-c1,\lambda;1\right)_{,i} \\ (\alpha_j,A_j;a_j)_p(e-c,\lambda;1) \\ \hline (e-c+1,\lambda;1)\ {}_j(\beta_j,B_j;1)_{m.m-1} \\ {}_j(\beta_j,B_j;b_j)_q(e+a-2c-1,2\lambda;1) \\ \left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right) \end{matrix}\right\}\right)\right)}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx=1$

Where

$$f(x)=F_1^2\left(\alpha, 2-\alpha, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$$

$$\bar{I}^{m,n}_{p,q}\left(\left\{z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}{}_j(\alpha_j,A_j;a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\end{matrix}\right.\right\}\right)$$

Sixth Integral

$$x^{c-1}(1-x)^{c-e-1}[.1+\alpha x+\beta(1-x)]^{-2c+e}F_1^2\left(a,-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$$

$$\int_0^1 \bar{I}^{m,n}_{p,q}\left(\left\{z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}{}_j(\alpha_j,A_j;a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\end{matrix}\right.\right\}dx\right.$$

$$=\frac{\Gamma(e)\Gamma(a-1)}{2^{2a+1}(1+\alpha)^c(1+\beta)^{c-e}\Gamma(e-a)}*U\bar{I}^{m,n+3}_{p+3,,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(1-c+e,\lambda;1),.(1-c,\lambda;1).\\ \left(1-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\\ (1-2c+e+a,2\lambda;1)\left(1-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right)\end{matrix}\right.\right\}\right)-$$

$$V.\bar{I}^{m,n+1}_{p+3,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda;1),.(1-c,\lambda;1)\\ {}_j(\beta_j,B_j;1)_{m.m-1}\ {}_j(\beta_j,B_j;b_j)_q\\ (e+a-2c-1,2\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)\end{matrix}\right.\right\}\right)$$

……………………………………………………………………………….(5.3.6)

Provided ⅄>0, Re(c-e+1+⅄) $\min_{1\le j\le m}(\beta_j/B_j)>0$, Re(c+⅄) $\min_{1\le j\le m}(\beta_j/B_j)>0$ and Re(c-1/2e-1/2a+1/2+⅄) $\min_{1\le j\le m}(\beta_j/B_j)>0$, ø>0, $|arg(z)|<\theta_{\pi/2}$, where θ is same as given. Also the constants α and β such that none of the expressions 1+α, 1+β, [1+αx+β(1-x)], where 0≤x≤1, is not zero.

$$U=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)},V=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)}$$

Then by definition of probability distribution, we have from (5.3.6):

$$f(x)=\frac{\dfrac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(e-a)}*\left(U\bar{I}^{m+1,n+3}_{p+4,,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda;1),.(1-c,\lambda;1).\\ \left(\frac{1}{2e}+\frac{1}{2a},-c\,\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p(e-c,\lambda;1)\\ (e-c-1,\lambda;1)\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta_j,B_j;b_j)_q\\ (e+a-2c-1,,2\lambda;1)\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}-c,\lambda;1\right)\end{matrix}\right.\right\}\right)-V.\bar{I}^{m+1,n+2}_{p+2,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda;1),.(1-c,\lambda;1).\\ \left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}-c1,\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p(e-c,\lambda;1)\\ (e-c+1,\lambda;1)\ {}_j(\beta_j,B_j;1)_{m.m-1}\ {}_j(\beta_j,B_j;b_j)_q\\ (e+a-2c-1,2\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)\end{matrix}\right.\right\}\right)\right)}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx=1$

Where

$$f(x)=F_1^2\left(a,2-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{I}^{m,n}_{p,q}$$

$$\left\{ z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}} \middle| \frac{{}_{j}(\alpha_j, A_j; a_j)_p}{{}_{j}(\beta_j, B_j; 1)_{m.m+1}\ {}_{j}(\beta, B_j; b_j)_q} \right\}$$

Seventh Integral

$$x^{c-1}(1-x)^{c-e-1}[.1+\alpha x+\beta(1-x)]^{-2c+e} F_1^2\left(a, 1-\alpha, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$$

$$\int_0^1 \quad \bar{I}_{p,q}^{m,n}\left(\left\{ z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}} \middle| \frac{{}_{j}(\alpha_j,A_j;a_j)_p}{{}_{j}(\beta_j,B_j;1)_{m.m+1}\ {}_{j}(\beta,B_j;b_j)_q} \right\} dx\right.$$

$$=\frac{\Gamma(\mathrm{e})}{2^{2a}(1+\alpha)^{c}(1+\beta)^{c-e}\Gamma(\mathrm{e-a})} * \mathbf{U}$$

$$\bar{I}_{p+3,,q+2}^{m,n+3}\left(\left\{ \frac{z}{(1+\alpha)(1+\beta)} \middle| \begin{matrix} (1-c+e,\lambda;1),.(2-c,\lambda;1). \\ \left(1-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p \\ \hline {}_{j}(\beta_j,B_j;1)_{m.m+1}\ {}_{j}(\beta_j,B_j;b_j)_q \\ (1-2c+e+a,\lambda;1)\left(-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right) \end{matrix} \right\}\right. -$$

$$\mathrm{V}.\ \bar{I}_{p+3,q+2}^{m,n+3}\left(\left\{ \frac{z}{(1+\alpha)(1+\beta)} \middle| \begin{matrix} (e-c-1,\lambda;1),.(2-c,\lambda;1) \\ \left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p \\ \hline {}_{j}(\beta_j,B_j;1)_{m.m+1}\ {}_{j}(\beta_j,B_j;b_j)_q \\ (e+a-2c+1,2\lambda;1)\left(\frac{3}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right) \end{matrix} \right\}\right.$$

…………………………………………………………………………………..(5.3.7)

Provided $\lambda>0$, Re(c-e+1+λ) $\min_{1\le j\le m}(\beta_j/B_j) > 0$, Re(c+$\lambda$) $\min_{1\le j\le m}(\beta_j/B_j) > 0$ and Re(c-1/2e-1/2a+1/2+λ) $\min_{1\le j\le m}(\beta_j/B_j) > 0$, $\emptyset>0$, $|arg(z)| < \theta_{\pi/2}$, where θ is same as given. Also the constants α and β such that none of the expressions 1+α, 1+β, [1+αx+β(1-x)], where 0≤x≤1, is not zero.

$$\mathrm{U}=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)},\ \mathrm{V}=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)}$$

Then by definition of probability distribution, we have from (5.3.7): $f(x) =$

$$\frac{\dfrac{\Gamma(\mathrm{e})}{2^{2a}(1+\alpha)^{c}(1+\beta)^{c-e}\Gamma(\mathrm{e-a})} * \mathrm{U}\bar{I}_{p+3,,q+2}^{m,n+3}\left(\left\{ \frac{z}{(1+\alpha)(1+\beta)} \middle| \begin{matrix} (1-c+e,\lambda;1),.(2-c,\lambda;1). \\ \left(1-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p \\ \hline {}_{j}(\beta_j,B_j;1)_{m.m+1}\ {}_{j}(\beta_j,B_j;b_j)_q \\ (1-2c+e+a,\lambda;1)\left(-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right) \end{matrix} \right\}\right. - \mathrm{V}.\bar{I}_{p+3,q+2}^{m,n+3}\left(\left\{ \frac{z}{(1+\alpha)(1+\beta)} \middle| \begin{matrix} (e-c-1,\lambda;1),.(2-c,\lambda;1) \\ \left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p \\ \hline {}_{j}(\beta_j,B_j;1)_{m.m+1}\ {}_{j}(\beta_j,B_j;b_j)_q \\ (e+a-2c+1,2\lambda;1)\left(\frac{3}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right) \end{matrix} \right\}\right.}{\int_0^1 x^{c-1}(1-x)^{c-e-1}[.1+\alpha x+\beta(1-x)]^{-2c+e} dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx = 1$

Where

f(x) = $F_1^2\left(\alpha, 1-\alpha, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right) \bar{I}_{p,q}^{m,n}$

$$\left\{z \frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}} \middle| \frac{{}_j(\alpha_j, A_j; a_j)_p}{{}_j(\beta_j, B_j; 1)_{m.m+1}\ {}_j(\beta, B_j; b_j)_q}\right\}$$

5.4 PROOF OF THE INTEGRALS

In order to prove the integral (5.3.1), we proceed follows:

Denoting the left hand side of (5.3.1) by I, expressing the Ī-function by means of its contour integral as given, we have

$$I = \int_0^1 x^{c-1}(1-x)^{c-e}[1+\alpha x+\beta(1-x)]^{-2c+e-1} F_1^2\left(\alpha, 1-\alpha, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right) X\,(2\pi i)^{-1}\int \theta(s)\frac{x^{\lambda x}(1-x))^{\lambda x}}{[1+\alpha x+\beta(1-x)]^{2\lambda x}} Z^s \text{ds dx} \ldots\ldots\ldots\ldots(5.4.1)$$

Now, Change the order of Integration which is seen to be justified by the application of well-known De L Vallee Poussion's theorem, We have:

$$I = (2\pi i)^{-1}\int \theta(s)\, Z^s\{\int_0^1 x^{c+\lambda x-1}(1-x)^{c+\lambda x-e}[1+\alpha x+\beta(1-x)]^{-2c-2\lambda x+e-1} F_1^2\left(\alpha, 1-\alpha, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right) dx\,\}ds \ldots\ldots\ldots\ldots(5.4.2)$$

Now, if we evaluate the intetgral with help of known result (5.2.1), we have, after little simplification

$$I = \frac{\Gamma(\mathrm{e})\Gamma\left(\frac{1}{2\mathrm{e}}-\frac{1}{2\mathrm{a}}\right)}{2^{2x}(1+\alpha)^e(1+\beta)^{c-e+1}\Gamma(\mathrm{e}-\mathrm{a})\Gamma\left(\frac{1}{2\mathrm{e}}-\frac{1}{2\mathrm{a}}\right)}(2\pi i)^{-1}X$$

$$\int \theta(s)\frac{[\Gamma(\mathrm{c}+\lambda\mathrm{x})\Gamma(\mathrm{c}+\lambda\mathrm{x}-\mathrm{c}+1)\Gamma\left(c+\lambda x-\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)]}{(1+\alpha)^{\lambda x}(1+\beta)^{\lambda x}\Gamma(\mathrm{c}+\lambda\mathrm{x}+\frac{1}{2a}-\frac{1}{2e}+\frac{1}{2})\Gamma(2c+2\lambda x-a-c+1)]} Z^x ds \ldots\ldots\ldots(5.4.3)$$

On interpreting the result thus obtained with the help of definition of integral (1.5.7), We arrive the right hand side of (5.3.1).

In exactly the same manner, the results (5.3.2) to (5.3.7) can also be established with the help of the results (5.2.2.) to (5.2.7), respectively.

5.5. SPECIAL CASES

1. In (5.3.1) to (5.3.7) if take $a_j = 1$, j = n+1….., p, we get the following integral involving $\bar{H}$ – function introduced earlier by Inayat Hussain [68] and Gaur [2003].

First Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e}[1+\alpha x+\beta(1-x)]^{-2c+e-1} F_1^2\left(\alpha, 1-\alpha, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{H}_{p,q}^{m,n}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\middle|\frac{{}_j(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q}\right\}dx\right.$$

$$= \frac{\Gamma(\mathrm{e})\Gamma(\frac{1}{2e}-\frac{1}{2a})}{2^{2s}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma(\mathrm{e}-\mathrm{a})} *$$

$$\bar{H}_{p+3,q+2}^{m,n+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\frac{(e-c,\lambda;1),.(1-c,\lambda;1).(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1),{}_i(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\ (e+a-2c,2\lambda;1)(\frac{1}{2}+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1)}\right\}\right. \ldots\ldots\ldots(5.5.1)$$

Provided the condition easily obtaible from (5.3.1) are satisfied. Then by definition of probability distribution, we have (5.5.1):

$$f(x)=\frac{\frac{\Gamma(\mathrm{e})\Gamma\left(\frac{1}{2\mathrm{e}}-\frac{1}{2\mathrm{a}}\right)}{2^{2s}(1+\alpha)^{c}(1+\beta)^{c-e+1}\Gamma\left(\frac{1}{2\mathrm{e}}+\frac{1}{2\mathrm{a}}\right)\Gamma(\mathrm{e}-\mathrm{a})}*\bar{H}^{m,n+3}_{p+3,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{array}{c}(e-c,\lambda;1),.(1-c,\lambda;1).\\ (\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1)_{,i}(\alpha_j,A_j;\ a_j)_p\\ \hline {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\\ (e+a-2c,2\lambda;1)(\frac{1}{2}+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1)\end{array}\right.\right\}\right)}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx=1$

Where

$$\mathrm{f(x)}=F_1^2\left(\alpha,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{H}^{m,n}_{p,q}\left(\left\{z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\frac{{}_j(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q}\right.\right\}\right)$$

Second Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1}F_1^2\left(\alpha,2-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{H}^{m,n}_{p,q}\left(\left\{z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\frac{{}_j(\alpha_j,A_j;a_j)_n\ {}_{m+1}(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q}\right.\right\}\right)dx=$$

$$=\frac{\Gamma(\mathrm{e})\Gamma(a_{-1})}{2^{2a-1}(1+\alpha)^{c}(1+\beta)^{c-e+1}\Gamma(a)\Gamma(\mathrm{e}-\mathrm{a})}*\mathrm{U}$$

$$\bar{H}^{m,n+1}_{p+3,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{array}{c}(e-c,\lambda;1),.(2-c,\lambda;1)\\ .\left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;\ a_j)_p\\ \hline {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\\ (e+a-2c,2\lambda;1)\left(\frac{3}{2}+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)\end{array}\right.\right\}\right)-$$

$$\mathrm{V}.\ \bar{H}^{m,n+1}_{p+3,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{array}{c}(e-c,\lambda;1),.(2-c,\lambda;1).\\ \left(\frac{1}{2e}+\frac{1}{2a}-c,\lambda;1\right)_{,i}(\alpha_j,A_j;\ a_j)_p\\ \hline {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\\ (e+a-2c,2\lambda;1)\left(1-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right)\end{array}\right.\right\}\right)\ldots(5.5.2)$$

Provided the condition easily abatable from (5.3.2) are satisfied. Then by definition of probability distribution, we have (5.5.2): where

$$\mathrm{U}=\frac{\Gamma(\frac{1}{2e}-\frac{1}{2a})}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)},\quad \mathrm{V}=\frac{\Gamma(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2})}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)}$$

Then by definition of probability distribution, we have from (5.5.2):

=0

Elsewhere, $\int_0^1 f(x)dx=1$

Where

$$\mathrm{f(x)}=F_1^2\left(\alpha,2-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{H}^{m,n}_{p,q}\left(\left\{z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\frac{{}_j(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q}\right.\right\}\right)$$

Third Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1} F_1^2\left(a,-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{H}_{p,q}^{m,n}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\middle|\begin{matrix}{}_j(\alpha_j,A_j;a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\end{matrix}\right\}\right)dx$$

$$=\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(e-a)}*U$$

$$\bar{I}_{p+3,q+2}^{m,n+1}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\begin{matrix}(e-c,\lambda;1),.(2-c,\lambda;1).\left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\ (e+a-2c,2\lambda;1)\left(\frac{1}{2}+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)\end{matrix}\right\}\right)-$$

$$V.\ \bar{I}_{p+3,q+2}^{m,n+1}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\begin{matrix}(e-c,\lambda;1),.(2-c,\lambda;1).\left(\frac{1}{2e}+\frac{1}{2a}-c,\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\ (e+a-2c,2\lambda;1)\left(\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)\end{matrix}\right\}\right)\ldots\ldots(5.5.3)$$

Provided the condition easily abatable from (5.3.3) are satisfied. Then by definition of probability distribution, we have (5.5.3):

$$U=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)},V=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)}$$

Then by definition of probability distribution, we have from (5.5.3):

$$f(x)=\frac{\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(e-a)}*\ U\bar{I}_{p+3,q+2}^{m,n+1}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\begin{matrix}(e-c,\lambda;1),.(2-c,\lambda;1).\left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\ (e+a-2c,2\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right)\end{matrix}\right\}\right)-V.\bar{I}_{p+3,q+2}^{m,n+1}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\begin{matrix}(e-c,\lambda;1),.(2-c,\lambda;1).\left(\frac{1}{2e}+\frac{1}{2a}-c,\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\ (e+a-2c,2\lambda;1)\left(\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)\end{matrix}\right\}\right)}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx=1$

Where

$$f(x)=F_1^2\left(a,2-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{I}_{p,q}^{m,n}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\middle|\begin{matrix}{}_j(\alpha_j,A_j;a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\end{matrix}\right\}\right)$$

Fourth Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1} F_1^2\left(a,-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{I}_{p,q}^{m,n}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\middle|\begin{matrix}{}_j(\alpha_j,A_j;a_j)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q\end{matrix}\right\}\right)dx$$

$$=\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(e-a)}*U$$

$$\overline{H}^{m+1,n+2}_{p+2,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda;1),.(1-c,\lambda;1).\\ \left(\frac{1}{2e}+\frac{1}{2a'}-c\ \lambda;1\right)_{,i}(\alpha_j,A_j;\ a_j)_p(e-c,\lambda;1)\\ \hline (e-c-1,\lambda;1)\ _j(\beta_j,B_j;1)_{m.m+1}\ _j(\beta_j,B_j;b_j)_q\\ (e+a-2c-1,,2\lambda;1)\left(1-c+\frac{1}{2e}-\frac{1}{2a'},\lambda;1\right)\end{matrix}\right.\right\}\right)-$$

$$V.\ \overline{H}^{m+1,n+2}_{p+2,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c,\lambda;1),.(1-c,\lambda;1).\\ \left(\frac{1}{2e}+\frac{1}{2a}-c-1,\lambda;1\right)_{,i}(\alpha_j,A_j;\ a_j)_p(e-c,\lambda;1)\\ \hline (e-c,\lambda;1)\ _j(\beta_j,B_j;1)_{m.m-1}\ _j(\beta_j,B_j;b_j)_q\\ (e+a-2c,1\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a'},\lambda;1\right)\end{matrix}\right.\right\}\right)(5.5.4)$$

Provided the condition easily obtaible from (5.3.4) are satisfied. Then by definition of probability distribution, we have (5.5.4):

$$U=\frac{\Gamma(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2})}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)},V=\frac{\Gamma(\frac{1}{2e}-\frac{1}{2a})}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)}$$

Then by definition of probability distribution, we have from (5.5.4):

$$f(x)=\frac{\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(\alpha)\Gamma(e-a)}\left(*\,U\overline{H}^{m+1,n+2}_{p+2,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda;1),.(1-c,\lambda;1).\\ \left(\frac{1}{2e}+\frac{1}{2a'},-c\ \lambda;1\right)_{,i}(\alpha_j,A_j;\ a_j)_p(e-c,\ \lambda;1)\\ \hline (e-c-1,\lambda;1)\ _j(\beta_j,B_j;1)_{m.m+1}\ _j(\beta_j,B_j;b_j)_q\\ (e+a-2c-1,,2\lambda;1)\left(1-c+\frac{1}{2e}-\frac{1}{2a'},\lambda;1\right)\end{matrix}\right.\right\}\right)-V.\overline{H}^{m+1,n+2}_{p+2,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c,\lambda;1),.(1-c,\lambda;1)\\ .\left(\frac{1}{2e}+\frac{1}{2a}-c-1,\lambda;1\right)_{,i}(\alpha_j,A_j;\ a_j)_p(e-c,\lambda;1)\\ \hline (e-c,\lambda;1)\ _j(\beta_j,B_j;1)_{m.m-1}\ _j(\beta_j,B_j;b_j)_q\\ (e+a-2c,1\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a'},\lambda;1\right)\end{matrix}\right.\right\}\right)\right)}{\int_0^1 x^{c-1}\,(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}\,dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx = 1$

Where

$$f(x) = F_1^2\left(a,2-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{I}^{m,n}_{p,q}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}_j(\alpha_j,A_j;a_j)_p\\ \hline _j(\beta_j,B_j;1)_{m.m+1}\ _j(\beta,B_j;b_j)_q\end{matrix}\right.\right\}\right)$$

Fifth Integral

$$\int_0^1 x^{c-1}\,(1-x)^{c-e+1}[.1+\alpha x+\beta(1-x)]^{-2c+e-2}F_1^2\left(a,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\overline{H}^{m,n}_{p,q}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}_j(\alpha_j,A_j;a_j)_p\\ \hline _j(\beta_j,B_j;1)_{m.m+1}\ _j(\beta,B_j;b_j)_q\end{matrix}\right.\right\}\right)dx$$

$$\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(\alpha)\Gamma(e-a)}$$

$$=*U\bar{I}^{m+1,n+3}_{p+4,,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\frac{(e-c-1,\lambda;1),.(1-c,\lambda;1).\left(\frac{1}{2e}+\frac{1}{2a},-c\,\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p(e-c,\lambda;1)}{(e-c-1,\lambda;1)\,_j(\beta_j,B_j;1)_{m.m+1}\,_j(\beta_j,B_j;b_j)_q(e+a-2c-1,,2\lambda;1)\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}-c,\lambda;1\right)}\right\}\right)-$$

$$V.\,\bar{H}^{m+1,n+2}_{p+2,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\frac{(e-c-1,\lambda;1),.(1-c,\lambda;1).\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}-c1,\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p(e-c,\lambda;1)}{(e-c+1,\lambda;1)\,_j(\beta_j,B_j;1)_{m.m-1}\,_j(\beta_j,B_j;b_j)_q(e+a-2c-1,2\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)}\right\}\right)$$

……………………………………………………………………………………..(5.5.5)

Provided the condition easily obtaible from (5.3.5) are satisfied. Then by definition of probability distribution, we have (5.5.5):

$$U=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)},\quad V=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)}$$

Then by definition of probability distribution, we have from (5.5.5):

$$f(x)=\frac{\dfrac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(\alpha)\Gamma(e-a)}\left(\left(*\,U\bar{H}^{m+1,n+3}_{p+4,,q+3}\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\frac{(e-c-1,\lambda;1),.(1-c,\lambda;1).\left(\frac{1}{2e}+\frac{1}{2a},-c\,\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p(e-c,\lambda;1)}{(e-c-1,\lambda;1)\,_j(\beta_j,B_j;1)_{m.m+1}\,_j(\beta_j,B_j;b_j)_q(e+a-2c-1,,2\lambda;1)\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}-c,\lambda;1\right)}\right\}\right)\right)-V.\bar{H}^{m+1,n+2}_{p+2,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\frac{(e-c-1,\lambda;1),.(1-c,\lambda;1).\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}-c1,\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p(e-c,\lambda;1)}{(e-c+1,\lambda;1)\,_j(\beta_j,B_j;1)_{m.m-1}\,_j(\beta_j,B_j;b_j)_q(e+a-2c-1,2\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)}\right\}\right.}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}\,dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx=1$

Where

$$f(x)=F_1^2\left(\alpha,2-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$$

$$\bar{I}^{m,n}_{p,q}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\middle|\frac{_j(\alpha_j,A_j;a_j)_p}{_j(\beta_j,B_j;1)_{m.m+1}\,_j(\beta,B_j;b_j)_q}\right\}\right)$$

Sixth Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e-1}[1+\alpha x+\beta(1-x)]^{-2c+e}F_1^2\left(a,-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right) \bar{\mathrm{I}}_{p,q}^{m,n}\left(\left\{z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}{}_{j}^{\square}(\alpha_j,A_j;a_j)_p\\ {}_{j}^{\square}(\beta_j,B_j;1)_{m.m+1}\ {}_{j}^{\square}(\beta,B_j;b_j)_q\end{matrix}\right.\right\}dx\right.$$

$$=\frac{\Gamma(\mathrm{e})\Gamma(a-1)}{2^{2a+1}(1+\alpha)^c(1+\beta)^{c-e}\Gamma(\mathrm{e-a})}*U\bar{H}_{p+3,,q+2}^{m,n+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(1-c+e,\lambda;1),.(1-c,\lambda;1).\\ \left(1-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p\\ \hline {}_{j}^{\square}(\beta_j,B_j;1)_{m.m+1}\ {}_{j}^{\square}(\beta_j,B_j;b_j)_q\\ (1-2c+e+a,2\lambda;1)\left(1-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right)\end{matrix}\right.\right\}\right.-$$

$$\mathrm{V}.\bar{H}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda;1),.(1-c,\lambda;1)\\ \hline {}_{j}^{\square}(\beta_j,B_j;1)_{m.m-1}\ {}_{j}^{\square}(\beta_j,B_j;b_j)_q\\ (e+a-2c-1,2\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)\end{matrix}\right.\right\}\right)$$

……………………………………………………………………………………..(5.5.6)

Provided the condition easily obtaible from (5.3.6) are satisfied. Then by definition of probability distribution, we have (5.5.6):

$$\mathrm{U}=\frac{\Gamma\left(\frac{1}{2\mathrm{e}}-\frac{1}{2\mathrm{a}}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)},\ \mathrm{V}=\frac{\Gamma\left(\frac{1}{2\mathrm{e}}-\frac{1}{2\mathrm{a}}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)}$$

Then by definition of probability distribution, we have from (5.5.6):

$$f(x)=\frac{\frac{\Gamma(\mathrm{e})\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(\mathrm{e-a})}*\left(U\bar{H}_{p+4,,q+3}^{m+1,n+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda;1),.(1-c,\lambda;1).\\ \left(\frac{1}{2e}+\frac{1}{2a},-c\ \lambda;1\right)_{,i}(\alpha_j,A_j;\ a_j)_p(e-c,\ \lambda;1)\\ \hline (e-c-1,\lambda;1)\ {}_{j}^{\square}(\beta_j,B_j;1)_{m.m+1}\ {}_{j}^{\square}(\beta_j,B_j;b_j)_q\\ (e+a-2c-1,,2\lambda;1)\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}-c,\lambda;1\right)\end{matrix}\right.\right\}\right)-\mathrm{V}.\bar{H}_{p+2,q+3}^{m+1,n+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda;1),.(1-c,\lambda;1).\\ \left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}-c1,\lambda;1\right)_{,i}(\alpha_j,A_j;\ a_j)_p(e-c,\lambda;1)\\ \hline (e-c+1,\lambda;1)\ {}_{j}^{\square}(\beta_j,B_j;1)_{m.m-1}\ {}_{j}^{\square}(\beta_j,B_j;b_j)_q\\ (e+a-2c-1,2\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)\end{matrix}\right.\right\}\right)\right)}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx=1$

Where

$\mathrm{f(x)}=F_1^2\left(a,2-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{\mathrm{I}}_{p,q}^{m,n}$

$$\left\{z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\middle|\frac{{}_j(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q}\right\}$$

Seventh Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e-1}[.1+\alpha x+\beta(1-x)]^{-2c+e}F_1^2\left(a,1-\alpha,e;\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{H}_{p,q}^{m,n}\left(\left\{z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\middle|\frac{{}_j(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta,B_j;b_j)_q}\right\}dx\right.$$

$$=\frac{\Gamma(e)}{2^{2a}(1+\alpha)^{c}(1+\beta)^{c-e}\Gamma(e-a)}*U$$

$$\bar{H}_{p+3,,q+2}^{m,n+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\frac{(1-c+e,\lambda;1),.(2-c,\lambda;1).\ \left(1-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta_j,B_j;b_j)_q\ (1-2c+e+a,\lambda;1)\left(-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right)}\right\}\right.-$$

$$V.\ \bar{H}_{p+3,q+2}^{m,n+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\frac{(e-c-1,\lambda;1),.(2-c,\lambda;1)\ \left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta_j,B_j;b_j)_q\ (e+a-2c+1,2\lambda;1)\left(\frac{3}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)}\right\}\right.$$

……………………………………………………………………………………(5.5.7)

Provided the condition easily obtaible from (5.3.7) are satisfied. Then by definition of probability distribution, we have (5.5.7):

$$U=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)},V=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)}$$

Then by definition of probability distribution, we have from (5.5.7):

$$f(x)=\frac{\frac{\Gamma(e)}{2^{2a}(1+\alpha)^{c}(1+\beta)^{c-e}\Gamma(e-a)}*U\bar{H}_{p+3,,q+2}^{m,n+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\frac{(1-c+e,\lambda;1),.(2-c,\lambda;1).\ \left(1-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta_j,B_j;b_j)_q\ (1-2c+e+a,\lambda;1)\left(-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right)}\right\}\right.-V.\bar{H}_{p+3,q+2}^{m,n+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\middle|\frac{(e-c-1,\lambda;1),.(2-c,\lambda;1)\ \left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p}{{}_j(\beta_j,B_j;1)_{m.m+1}\ {}_j(\beta_j,B_j;b_j)_q\ (e+a-2c+1,2\lambda;1)\left(\frac{3}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1\right)}\right\}\right.}{\int_0^1 x^{c-1}(1-x)^{c-e-1}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx=1$

Where

f(x) = $F_1^2\left(a, 1-\alpha, e; \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right) \bar{I}_{p,q}^{m,n}$

$$\left\{ z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}} \middle| \begin{matrix} {}_j(\alpha_j, A_j; a_j)_p \\ {}_j(\beta_j, B_j; 1)_{m.m+1}\ {}_j(\beta, B_j; b_j)_q \end{matrix} \right\}$$

2. In (5.3.1) to (5.3.7) if take a_j = 1,….,n,and b_j =1, j=m+1,….., q, we get the following integral involving $\bar{H}$ – function introduced earlier by Fox[52].

First Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e}[1+\alpha x+\beta(1-x)]^{-2c+e-1} F_1^2\left(a, 1-\alpha, e; \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right) \bar{H}_{p,q}^{m,n}\left(\left\{ z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}} \middle| \begin{matrix} {}_j(\alpha_j, A_j; a_j)_p \\ {}_j(\beta_j, B_j; 1)_{m.m+1}\ {}_j(\beta, B_j; b_j)_q \end{matrix} \right\}\right) dx$$

$$= \frac{\Gamma(\mathrm{e})}{2^{2s}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma\Gamma(\mathrm{e}-\mathrm{a})} *$$

$$\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)} X \bar{H}_{p+3,q+2}^{m,n+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)} \middle| \begin{matrix} (e-c,\lambda;1),.(1-c,\lambda;1).(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda;1)_{,i}(\alpha_j, A_j; a_j)_p \\ {}_j(\beta_j, B_j; 1)_{m.m+1}\ {}_j(\beta, B_j; b_j)_q\ (e+a-2c, 2\lambda;1)(\frac{1}{2}+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;1) \end{matrix}\right\}\right) \ldots\ldots\ldots\ldots$$

.(5.5.8)

Provided the condition easily obtaible from (5.3.1) are satisfied. Then by definition of probability distribution, we have (5.5.8):

$$f(x) = \frac{\frac{\Gamma(\mathrm{e})}{2^{2s}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)\Gamma(\mathrm{e}-\mathrm{a})} * \frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)} \bar{H}_{p+3,q+2}^{m,n+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)} \middle| \begin{matrix} (e-c,\lambda),.\left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda\right). \\ (\alpha_j, A_j)_p \\ \hline {}_j(\beta_j, B_j)_q \\ (e+a-2c, 2\lambda)(\frac{1}{2}+\frac{1}{2e}-\frac{1}{2a}-c,\lambda;) \end{matrix}\right\}\right)}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1} dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx = 1$

Where

f(x) = $F_1^2\left(a, 1-\alpha, e; \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right) \bar{H}_{p,q}^{m,n}\left(\left\{ z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}} \middle| \begin{matrix} {}_j(\alpha_j, A_j)_p \\ {}_j(\beta, B_j)_q \end{matrix} \right\}\right)$

Second Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e-1} F_1^2\left(a, 2-\alpha, e; \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right) \bar{H}_{p,q}^{m,n}\left(\left\{ z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}} \middle| \begin{matrix} {}_i(\alpha_j, A_j)_p \\ {}_j(\beta, B_j; b_j)_q \end{matrix} \right\}\right) dx =$$

$$= \frac{\Gamma(\mathrm{e})\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(\mathrm{e}-\mathrm{a})} * \{$$

$$\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)}\bar{H}^{m,n+1}_{p+3,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\frac{\begin{matrix}(e-c,\lambda),.(2-c,\lambda)\\ .\left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda\right)_{,i}(\alpha_j,A_j)_p\end{matrix}}{\begin{matrix}{}_j(\beta,B_j)_q\\ (e+a-2c,2\lambda)\left(\frac{3}{2}+\frac{1}{2e}-\frac{1}{2a}-c,\lambda\right)\end{matrix}}\right.\right\}\right)-$$

$$\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)}.\bar{H}^{m,n+1}_{p+3,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\frac{\begin{matrix}(e-c,\lambda),.(2-c,\lambda).\\ \left(\frac{1}{2e}+\frac{1}{2a}-c,\lambda\right)_{,i}(\alpha_j,A_j;)_p\end{matrix}}{\begin{matrix}{}_j(\beta_j,B_j)_q\\ (e+a-2c,2\lambda)\left(1-c+\frac{1}{2e}-\frac{1}{2a},\lambda\right)\end{matrix}}\right.\right\}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

………………..(5.5.9)

Provided the condition easily abatable from (5.3.2) are satisfied. Then by definition of probability distribution, we have (5.5.9): where

$$U=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)},\qquad V=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)}$$

Then by definition of probability distribution, we have from (5.5.9):

$$f(x)=\frac{\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(e-a)}\left(\begin{matrix}*\,U\bar{H}^{m,n+1}_{p+3,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\frac{\begin{matrix}(e-c,\lambda),.(2-c,\lambda).\\ \left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda\right)_{,i}(\alpha_j,A_j)_p\end{matrix}}{\begin{matrix}{}_j(\beta,B_j)_q\\ (e+a-2c,2\lambda)\left(\frac{3}{2}-c+\frac{1}{2e}-\frac{1}{2a},\lambda\right)\end{matrix}}\right.\right\}\right)\\ -V.\bar{H}^{m,n+1}_{p+3,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\frac{\begin{matrix}(e-c,\lambda),.(2-c,\lambda).\\ \left(\frac{1}{2e}+\frac{1}{2a}-c,\lambda\right)_{,i}(\alpha_j,A_j)_p\end{matrix}}{\begin{matrix}{}_j(\beta,B_j;b_j)_q\\ (e+a-2c,2\lambda)\left(1-c+\frac{1}{2e}-\frac{1}{2a},\lambda\right)\end{matrix}}\right.\right\}\right)\end{matrix}\right)}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx=1$

Where

$$f(x)=F_1^2\left(a,2-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{H}^{m,n}_{p,q}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\frac{{}_j(\alpha_j,A_j)_p}{{}_j(\beta,B_j;b_j)_q}\right.\right\}\right)$$

Third Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e+1}F_1^2\left(a,-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{H}^{m,n}_{p,q}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\frac{{}_j(\alpha_j,A_j)_p}{{}_j(\beta,B_j;)_q}\right.\right\}dx\right.$$

$$=\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(e-a)}*U$$

$$\bar{H}^{m,n+3}_{p+3,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\frac{(e-c,\lambda),.(2-c,\lambda).\left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda\right)_{,i}(\alpha_j,A_j)_p}{{}_j(\beta,B_j)_q\ (e+a-2c,2\lambda)\left(\frac{1}{2}+\frac{1}{2e}-\frac{1}{2a}-c,\lambda\right)}\right.\right\}\right)-$$

$$\text{V}.\, H^{m,n+1}_{p+3,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c,\lambda),.(2-c,\lambda).\left(\frac{1}{2e}+\frac{1}{2a}-c,\lambda\right)_{,i}(\alpha_j,A_j)_p\\ {}_j(\beta,B_j)_q\,(e+a-2c,2\lambda)\left(\frac{1}{2e}-\frac{1}{2a}-c,\lambda\right)\end{matrix}\right.\right\}\right)\ldots\ldots(5.5.10)$$

Provided the condition easily abatable from (5.3.3) are satisfied. Then by definition of probability distribution, we have (5.5.10):

$$\text{U}=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)},\text{V}=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)}$$

Then by definition of probability distribution, we have from (5.5.10):

$$f(x)=\frac{\dfrac{\Gamma(\mathrm{e})}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(\mathrm{e}-\mathrm{a})}*\begin{matrix}\text{U}\bar{I}^{m,n+3}_{p+3,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c,\lambda),.(2-c,\lambda).\left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda\right)_{,i}(\alpha_j,A_j)_p\\ {}_j(\beta,B_j)_q\,(e+a-2c,2\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a},\lambda\right)\end{matrix}\right.\right\}\right)\\ +\text{V}.\bar{I}^{m,n+1}_{p+3,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c,\lambda;1),.(2-c,\lambda;1).\left(\frac{1}{2e}+\frac{1}{2a}-c,\lambda\right)_{,i}(\alpha_j,A_j;)_p\\ {}_j(\beta_j,B_j;1)_{m.m+1}\;{}_j(\beta,B_j;b_j)_q\,(e+a-2c,2\lambda)\left(\frac{1}{2e}-\frac{1}{2a}-c,\lambda\right)\end{matrix}\right.\right\}\right)\end{matrix}}{\int_0^1 x^{c-1}\,(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx=1$

Where

$$\text{f(x)}=F_1^2\left(a,-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}{}_j(\alpha_j,A_j a_j)_p\\ {}_j(\beta,B_j)_q\end{matrix}\right.\right\}\right)$$

Fourth Integral

$$\int_0^1\int_0^1 x^{c-1}\,(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e+2}F_1^2\left(a,2-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{H}^{m,n}_{p,q}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}{}_j(\alpha_j,A_j)_p\\ {}_j(\beta,B_j;)_q\end{matrix}\right.\right\}\right)dx$$

$$=\frac{\Gamma(\mathrm{e})\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(\mathrm{e}-\mathrm{a})}*\text{U}$$

$$\bar{H}^{m+1,n+2}_{p+2,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda),.(1-c,\lambda).\\ \left(\frac{1}{2e}+\frac{1}{2a},-c\,\lambda\right)_{,i}(\alpha_j,A_j;)_p(e-c,\lambda)\\ (e-c-1,\lambda)\;{}_j(\beta_j,B_j)_q\\ (e+a-2c-1,,2\lambda)\left(1-c+\frac{1}{2e}-\frac{1}{2a},\lambda\right)\end{matrix}\right.\right\}\right)-$$

$$\text{V}.\,\bar{H}^{m+1,n+2}_{p+2,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c,\lambda),.(1-c,\lambda).\\ \left(\frac{1}{2e}+\frac{1}{2a}-c-1,\lambda;1\right)_{,i}(\alpha_j,A_j;a_j)_p(e-c,\lambda)\\ (e-c,\lambda;1)\;{}_j(\beta_j,B_j;1)_{m.m-1}\;{}_j(\beta_j,B_j;b_j)_q\\ (e+a-2c,1\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right)\end{matrix}\right.\right\}\right)(5.5.11)$$

Provided the condition easily obtaible from (5.3.4) are satisfied. Then by definition of probability distribution, we have (5.5.11):

$$\text{U}=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)},\text{V}=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)}$$

Then by definition of probability distribution, we have from (5.5.11):

$$f(x)=\frac{\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^{c}(1+\beta)^{c-e+1}\Gamma(a)\Gamma(e-a)} * U\bar{H}^{m+1,n+2}_{p+2,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda),.(1-c,\lambda).\\ \left(\frac{1}{2e}+\frac{1}{2a},-c\,\lambda\right)_{,i}(\alpha_j,A_j)_p(e-c,\lambda)\\ \hline (e-c-1,\lambda)\ {}_j(\beta_j,B_j;b_j)_q\\ (e+a-2c-1,,2\lambda)\left(1-c+\frac{1}{2e}-\frac{1}{2a},\lambda\right)\end{matrix}\right.\right\}\right)-V.\bar{H}^{m+1,n+2}_{p+2,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c,\lambda),.(1-c,\lambda)\\ .\left(\frac{1}{2e}+\frac{1}{2a}-c-1,\lambda\right)_{,i}(\alpha_j,A_j)_p(e-c,\lambda)\\ \hline (e-c,\lambda)\ {}_j(\beta_j,B_j)_q\\ (e+a-2-1,2\lambda;1)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a},\lambda;1\right)\end{matrix}\right.\right\}\right)}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx=1$

Where

$$f(x)=F_1^2\left(a,2-\alpha,e;\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{H}^{m,n}_{p,q}\left(\left\{z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}{}_j(\alpha_j,A_j)_p\\ {}_j(\beta,B_j)_q\end{matrix}\right.\right\}\right)$$

Fifth Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e+1}[.1+\alpha x+\beta(1-x)]^{-2c+e-2}F_1^2\left(a,1-\alpha,e;\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{H}^{m,n}_{p,q}\left(\left\{z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}{}_j(\alpha_j,A_j)_p\\ {}_j(\beta,B_j)_q\end{matrix}\right.\right\}dx\right.$$

$$=\frac{\Gamma(e)}{2^{2a}(1+\alpha)^{c}(1+\beta)^{c-e+2}\Gamma(e-a)}*U$$

$$\bar{H}^{m+1,n+3}_{p+4,,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda),.(1-c,\lambda).\\ \left(\frac{1}{2e}+\frac{1}{2a},-c\,\lambda\right)_{,i}(\alpha_j,A_j)_p(e-c,\lambda)\\ \hline (e-c+1,\lambda)\\ {}_j(\beta_j,B_j)_q(e+a-2c-1,2\lambda)\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}-c,\lambda\right)\end{matrix}\right.\right\}\right)-$$

$$V.\bar{H}^{m+1,n+2}_{p+2,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda),(1-c,\lambda).\\ \left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}-c1,\lambda\right)_{,i}(\alpha_j,A_j)_p(e-c,\lambda)\\ \hline (e-c+1,\lambda)\ {}_j(\beta_j,B_j)_q\\ (e+a-2c-1,2\lambda)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda\right)\end{matrix}\right.\right\}\right)$$

……………………………………………………………………………..(5.5.12)

Provided the condition easily obtaible from (5.3.5) are satisfied. Then by definition of probability distribution, we have (5.5.12):

$$U=\frac{\Gamma(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2})}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)},V=\frac{\Gamma(\frac{1}{2e}-\frac{1}{2a})}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)}$$

Then by definition of probability distribution, we have from (5.5.5):

$$f(x)=\frac{\frac{\Gamma(e)}{2^{2a-1}(1+\alpha)^{c}(1+\beta)^{c-e+1}\Gamma(e-a)} * U\bar{H}^{m+1,n+3}_{p+4,,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda),(1-c,\lambda).\\ \left(\frac{1}{2e}+\frac{1}{2a},-c\lambda\right)_{,i}(\alpha_j,A_j)_p(e-c,\lambda)\\ \hline (e-c-1,\lambda)\ {}_j(\beta_j,B_j)_q\\ (e+a-2c-1,,2\lambda)\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}-c,\lambda\right)\end{matrix}\right.\right\}\right) - V.\bar{H}^{m+1,n+2}_{p+2,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda),(1-c,\lambda).\\ \left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}-c1,\lambda\right)_{,i}\\ (\alpha_j,A_j)_p(e-c,\lambda)\\ \hline (e-c+1,\lambda)\\ {}_j(\beta_j,B_j)_q\ (e+a-2c-1,2\lambda)\\ \left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda\right)\end{matrix}\right.\right\}\right)}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx=1$

Where

$$f(x)=F_1^2\left(a,2-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)$$

$$\bar{H}^{m,n}_{p,q}\left(\left\{z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}{}_j(\alpha_j,A_j)_p\\ {}_j(\beta,B_j)_q\end{matrix}\right.\right\}\right)$$

Sixth Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e-1}[1+\alpha x+\beta(1-x)]^{-2c+e}F_1^2\left(a,-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{H}^{m,n}_{p,q}\left(\left\{z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}{}_j(\alpha_j,A_j)_p\\ {}_j(\beta,B_j)_q\end{matrix}\right.\right\}\right)dx$$

$$=\frac{\Gamma(e)}{2^{2a+1}(1+\alpha)^{c}(1+\beta)^{c-e}\Gamma(e-a)}*U\,\bar{H}^{m,n+3}_{p+3,,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(1-c+e,\lambda),.(1-c,\lambda).\\ \left(1-c+\frac{1}{2e}+\frac{1}{2a},\lambda\right)_{,i}(\alpha_j,A_j)_p\\ \hline {}_j(\beta_j,B_j)_q\\ (1-2c+e+a,2\lambda)\left(1-c+\frac{1}{2e}-\frac{1}{2a},\lambda\right)\end{matrix}\right.\right\}\right)-$$

$$V.\bar{H}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda),.(1-c,\lambda)\\ \hline {}_j(\beta_j,B_j)_q\\ (e+a-2c-1,2\lambda)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda\right)\end{matrix}\right.\right\}\right)$$

…………………………………………………………………………………..(5.5.13)

Provided the condition easily obtaible from (5.3.6) are satisfied. Then by definition of probability distribution, we have (5.5.13):

$$U=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)},\ V=\frac{\Gamma\left(\frac{1}{2e}-\frac{1}{2a}\right)}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-1\right)}$$

Then by definition of probability distribution, we have from (5.5.13):

$$f(x)=\frac{\frac{\Gamma(e)\Gamma(a-1)}{2^{2a-1}(1+\alpha)^c(1+\beta)^{c-e+1}\Gamma(a)\Gamma(e-a)}*\left(U\bar{H}^{m+1,n+3}_{p+4,,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda),(1-c,\lambda).\\\left(\frac{1}{2e}+\frac{1}{2a},-c\,\lambda\right)_{,i}(\alpha_j,A_j;)_p(e-c,\ \lambda)\\\hline(e-c-1,\lambda)\ {}_j(\beta_j,B_j)_q\\(e+a-2c-1,,2\lambda)\left(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2}-c,\lambda\right)\end{matrix}\right.\right\}\right)-V.\bar{H}^{m+1,n+2}_{p+2,q+3}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda),(1-c,\lambda).\\\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}-c1,\lambda\right)_{,i}(\alpha_j,A_j)_p(e-c,\lambda)\\\hline(e-c+1,\lambda)\ {}_j(\beta_j,B_j)_q\\(e+a-2c-1,2\lambda)\left(\frac{1}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda\right)\end{matrix}\right.\right\}\right)\right)}{\int_0^1 x^{c-1}(1-x)^{c-e}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx = 1$

Where

$$f(x) = F_1^2\left(a,2-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{H}^{m,n}_{p,q}\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}{}_j(\alpha_j,A_j)_p\\{}_j(\beta,B_j)_q\end{matrix}\right.\right\}$$

Seventh Integral

$$\int_0^1 x^{c-1}(1-x)^{c-e-1}[.1+\alpha x+\beta(1-x)]^{-2c+e}F_1^2\left(a,1-\alpha,e:\frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\bar{H}^{m,n}_{p,q}\left(\left\{z\frac{x^\lambda(1-x)^\lambda}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{matrix}{}_j(\alpha_j,A_j)_p\\{}_j(\beta,B_j)_q\end{matrix}\right.\right\}dx\right.$$

$$=\frac{\Gamma(e)}{2^{2a}(1+\alpha)^c(1+\beta)^{c-e}\Gamma(e-a)}*U\bar{H}^{m,n+3}_{p+3,,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(1-c+e,\lambda),.(2-c,\lambda).\\\left(1-c+\frac{1}{2e}+\frac{1}{2a},\lambda\right)_{,i}(\alpha_j,A_j)_p\\\hline{}_j(\beta_j,B_j)_q\\(1-2c+e+a,\lambda)\left(-c+\frac{1}{2e}-\frac{1}{2a},\lambda\right)\end{matrix}\right.\right\}\right)$$

$$-V.\ \bar{H}^{m,n+3}_{p+3,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{matrix}(e-c-1,\lambda),.(2-c,\lambda)\\\left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda\right)_{,i}(\alpha_j,A_j)_p\\\hline{}_j(\beta_j,B_j)_q\\(e+a-2c+1,2\lambda)\left(\frac{3}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda\right)\end{matrix}\right.\right\}\right)$$

……………………………………………………………………………………..(5.5.14)

Provided the condition easily obtaible from (5.3.7) are satisfied. Then by definition of probability distribution, we have (5.5.14):

$$U=\frac{\Gamma(\frac{1}{2e}-\frac{1}{2a}+\frac{1}{2})}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}-\frac{1}{2}\right)},V=\frac{\Gamma(\frac{1}{2e}-\frac{1}{2a})}{\Gamma\left(\frac{1}{2e}+\frac{1}{2a}\right)}$$

Then by definition of probability distribution, we have from (5.5.14):

$$f(x)=\frac{\frac{\Gamma(e)}{2^{2a}(1+\alpha)^{c}(1+\beta)^{c-e}\Gamma(e-a)} * U\overline{H}^{m,n+3}_{p+3,,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{array}{c}(1-c+e,\lambda),.(2-c,\lambda).\\ \left(1-c+\frac{1}{2e}+\frac{1}{2a},\lambda\right)_{,i}(\alpha_j,A_j)_p\\ \hline {}_j(\beta_j,B_j)_q\\ (1-2c+e+a,\lambda)\left(-c+\frac{1}{2e}-\frac{1}{2a},\lambda\right)\end{array}\right.\right\}-\text{v}.\overline{H}^{m,n+3}_{p+3,q+2}\left(\left\{\frac{z}{(1+\alpha)(1+\beta)}\left|\begin{array}{c}(e-c-1,\lambda),(2-c,\lambda)\\ \left(\frac{1}{2}-c+\frac{1}{2e}+\frac{1}{2a},\lambda\right)_{,i}(\alpha_j,A_j;)_p\\ \hline {}_j(\beta_j,B_j)_q\\ (e+a-2c+1,2\lambda)\left(\frac{3}{2}-c+\frac{1}{2e}-\frac{1}{2a}-c,\lambda\right)\end{array}\right.\right\}}{\int_0^1 x^{c-1}(1-x)^{c-e-1}[.1+\alpha x+\beta(1-x)]^{-2c+e}dx}$$

=0

Elsewhere, $\int_0^1 f(x)dx = 1$

Where

$$f(x) = F_1^2\left(a, 1-\alpha, e: \frac{(1+\alpha)x}{1+\alpha x+\beta(1-x)}\right)\overline{H}^{m,n}_{p,q}\left\{z\frac{x^{\lambda}(1-x)^{\lambda}}{[1+\alpha x+\beta(1-x)]^{2\lambda}}\left|\begin{array}{c}{}_j(\alpha_j,A_j)_p\\ \hline {}_j(\beta,B_j)_q\end{array}\right.\right\}$$

Similarly, other result can also be obtained.

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