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Dynamic Linear Model: A Review with Model

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INTRODUCTION

We provide a novel Bayesian modelling framework that may be used for both functional and time series data. Even though it has a wide range of applications, the methodology focuses on the difficult cases in which

(1) Functional data demonstrate additional dependence, such as time dependence or contemporaneous dependence;

(2) Functional or time series data demonstrate local features, such as jumps or rapidly changing smoothness; and

(3) A time series of functional data is observed sparsely or irregularly with non-negligible measurement error.

The use of the dynamic linear model (DLM) framework in novel settings to build highly effective Gibbs sampling algorithms is a feature that is common to all of the suggested approaches. This feature serves as a unifying factor. In order to model dependent functional data, we adapt DLMs that were designed for modelling multivariate time series data to the functional data situation. We then establish a smooth, time-invariant functional foundation for the functional observations. The model that was suggested allows for flexible modelling of complicated dependency structures among the functional data. These structures include temporal dependence, contemporaneous dependence, stochastic volatility, and covariates, among others. We apply the model to the data of yield curves for many economies as well as the local field potential of the brains of rats. We present an entirely new family of dynamic shrinkage processes as a means of doing locally adaptive Bayesian time series and regression analysis. We enable the local scale parameters to rely on the history of the shrinkage process in order to extend a large class of well-known global-local shrinkage priors, such as the horseshoe prior, to the dynamic setting. This allows us to make the priors applicable to a wider range of shrinkage problems. We show that the resultant processes inherit beneficial shrinkage behaviour from the non-dynamic analogize, but they also give extra locally adaptive shrinkage qualities. This is something that we verify. Using extensive simulations, a Bayesian trend filtering model for irregular curve-fitting of CPU usage data, and an adaptive time-varying parameter regression model, which we employ to study the dynamic relevance of the factors in the Fama-French asset pricing model, we demonstrate the substantial empirical gains that can be obtained from the proposed dynamic shrinkage processes. These gains can be attributed to the proposed dynamic shrinkage processes. Finally, we present a hierarchical functional autoregressive (FAR) model with Gaussian process innovations for the forecasting and inference of sparsely or irregularly sampled functional time series data. This model is intended to be used with sparsely sampled or irregularly sampled data. We demonstrate that the suggested model maintains its validity when the Gaussian assumption is relaxed by proving that its finite-sample forecasting and interpolation optimality characteristics are optimal. We put the suggested methodologies to use in order to provide forecasts of daily nominal and real yield curves that are very competitive. The Bayesian modelling approach is shown here for both functional and time series data. The approaches have a wide range of applications for (dependent) functional and time series data, but the following difficult scenarios, for which the existing methods are inadequate, are the primary emphasis of this chapter.

Uses of DLM

The use of the dynamic linear model (DLM) framework in new settings to generate interpretable models and computationally efficient MCMC sampling algorithms is a unifying aspect of the suggested methodologies. This framework is used to construct interpretable models and MCMC sampling algorithms. In particular, we create very efficient Gibbs sampling algorithms that rely upon previously developed DLM sampling components for large blocks of parameters (for example, Rue, 2001; Durbin and Koopman, 2002). These methods may be found in Rue, 2001; Durbin and Koopman, 2002. Novel applications of DLMs include dynamic shrinkage processes, functional autoregressive models, functional dynamic component models, and Bayesian trend filtering models. Importantly, the Bayesian framework allows for joint estimate of the parameters of model 1 and gives accurate inference (up to MCMC error) on individual parameters. This is a significant benefit. Important applications such as multi-economy interest rate modelling, nominal and real yield curve forecasting, dynamic extensions of the Fama-French asset pricing model, irregular curve-fitting of CPU usage data, and local field potential brain signals in rats served as inspiration for the proposed methodology. The approaches are tested by extensive simulations, and the findings are favourable when compared to alternative estimators that are considered to be state-of-the-art. We extend DLMs for multivariate time series to the functional data context in Chapter 2, wherein we offer a Bayesian model for multivariate, dependent functional data. In this model, we include data that are dependent on many variables. Additionally, Bayesian spline theory is developed by us within the context of a more generalised restricted optimisation framework. The approaches that have been developed locate a smooth and interpretable time-invariant functional foundation for the functional data. We analyse local field potential brain signals in rats, for which we build a multivariate functional time series approach for multivariate time-frequency analysis. We also utilise the technology to explore the interactions of multieconomy yield curves during the current global crisis. In Chapter 3, we provide an original class of dynamic shrinkage processes for the Bayesian analysis of time series and regression data. We enable the local scale parameters to rely on the history of the shrinkage process in order to extend a large class of well-known global-local shrinkage priors, such as the horseshoe prior, to the dynamic setting. This allows us to make the priors applicable to a wider range of shrinkage problems. We show that the resultant processes inherit beneficial shrinkage behaviour from the non-dynamic analogize, but they also give extra locally adaptive shrinkage qualities. This is something that we verify. The dynamic shrinkage processes that have been presented have a wide range of potential applications, in particular within the family of dynamic linear models. We construct a very effective Gibbs sampling algorithm by representing dynamic shrinkage processes on the log scale. This allows us to adopt successful approaches from stochastic volatility modelling. Additionally, we offer a Polya-Gamma scale' mixed representation. We use the proposed processes to produce superior Bayesian trend filtering estimates and posterior credible intervals for irregular curve-fitting of minute-by-minute Twitter CPU usage data. Additionally, we develop an adaptive time-varying parameter regression model in order to evaluate the efficacy of the Fama-French five-factor asset pricing model with momentum added as a sixth factor. In Chapter 4, we construct a hierarchical Gaussian process model for the purpose of making predictions and drawing inferences from functional time series data. Our methodology, in contrast to other approaches, is particularly useful for curves that have been sampled in an insufficient or inconsistent manner, as well as for curves that have been sampled with a significant amount of measurement error. The latent process is dynamically represented as a functional auto regression (FAR) with Gaussian process innovations, with extensions for FAR(p) models with model averaging across the lag p. p is the number of lags used in the analysis. We offer a completely nonparametric dynamic functional factor model for the dynamic innovation process. This model has greater applicability and enhanced computing efficiency in comparison to the more traditional Gaussian process models. We demonstrate that the suggested model maintains its validity when the Gaussian assumption is relaxed by proving that its finite-sample forecasting and interpolation optimality characteristics are optimal. Extensive simulations show that the autoregressive surface offers significant gains in terms of both its efficacy as a forecasting tool and its ability to recover from past errors in comparison to other approaches, particularly when used to sparse designs. We put the suggested strategies to use by utilising daily data from the United States to anticipate nominal and real yield curves. Although real yields are seen less frequently than nominal yields, the suggested strategies are very competitive in both the laboratory and the field contexts.

Historical Background of DLM

The hierarchical dynamic linear model (DLM) architecture that Gamerman and Migon (1993) and West and Harrison (1997) developed for modelling multivariate time series was extended to the functional data context so that it could be used to study MFTS. We extend Bayesian spline theory to a more broad restricted optimisation framework, which we then use for the purpose of parameter identifiability. This results in function estimates that are smooth, flexible, and optimum. Our requirements are made explicit in the posterior distribution by the use of proper conditioning of the typical Bayesian spline posterior distribution, and the mean of the posterior distribution that corresponds to our constraints is the solution to an appropriate optimisation problem. To collect samples from the joint posterior distribution, which enables precise inference for any parameters of interest (up to MCMC error), we develop an efficient Gibbs sampler and use these samples to test various hypotheses. The hierarchical Bayesian multivariate functional dynamic linear model that was suggested has higher application and utility than other models that are comparable. It is able to include application-specific prior information and allows flexible modelling of complicated dependency structures among the functional observations, such as temporal dependence, contemporaneous dependence, stochastic volatility, covariates, and change points.

DLM Model

Assuming $\mathcal{T}_o \subseteq \mathcal{T}_e$, let Z_t be the mt × M ncidence matrix that identifies the observations points observed at time t, i.e., $(\tau_{1,t}, \ldots, \tau_{m_t,t})' = \boldsymbol{Z}_t(\tau_1, \ldots, \tau_M)'$. The hierarchical model may be rewritten as a dynamic linear model (DLM; West and Harrison, 1997) in t:

$$\begin{cases} \boldsymbol{y}_t = \boldsymbol{Z}_t\boldsymbol{\mu} + \boldsymbol{Z}_t\boldsymbol{\mu}_t + \boldsymbol{\nu}_t, & [\boldsymbol{\nu}_t|\sigma_\nu^2] \overset{indep}{\sim} N(\boldsymbol{0}, \sigma_\nu^2\boldsymbol{I}_{m_t}) \text{ for } t = 1, \ldots, T, \\ \boldsymbol{\mu}_t = \boldsymbol{\Psi}\boldsymbol{Q}\boldsymbol{\mu}_{t-1} + \boldsymbol{\epsilon}_t, & [\boldsymbol{\epsilon}_t|\boldsymbol{K}_\epsilon] \overset{indep}{\sim} N(\boldsymbol{0}, \boldsymbol{K}_\epsilon) \text{ for } t = 2, \ldots, T, \\ \boldsymbol{\mu}_1 \sim N(\boldsymbol{0}, \boldsymbol{K}_\epsilon), \end{cases}$$

where $\boldsymbol{y}_t = (y_{1,t}, \ldots, y_{m_t,t})'$, $\boldsymbol{\mu} = (\mu(\tau_1), \ldots, \mu(\tau_M))'$, $\boldsymbol{\Psi} = \{\psi(\tau_i, \tau_k)\}_{i,k=1}^M$, and $\boldsymbol{K}_\epsilon = \{K_\epsilon(\tau_i, \tau_k)\}_{i,k=1}^M$. Model can be extended for multiple lags to the FAR(p) model by replacing the second level with $\boldsymbol{\mu}_t = \sum_{\ell=1}^p \boldsymbol{\Psi}_\ell\boldsymbol{Q}\boldsymbol{\mu}_{t-\ell} + \boldsymbol{\epsilon}_t$ for $\boldsymbol{\Psi}_\ell = \{\psi_\ell(\tau_i, \tau_k)\}_{i,k=1}^M$. The DLM formulation of the FAR(p) is helpful for MCMC sampling since efficient samplers exist for the vector-valued state variables, t (for example, Durbin and Koopman, 2002). This makes the DLM formulation of the FAR(p) beneficial. The Gibbs sampling technique that was presented for model is a mild expansion of the classic DLM samplers. It samples the state vectors t, the measurement error variance 2 v, the innovation covariance K, and the unknown evolution matrix in an iterative fashion. Additionally, the DLM makes it easier to estimate and predict non-Bayesian parameters. One example of this is an EM approach for estimating the latent state variables t with the parameters 2, K, and (for example, Cressie and Wikle, 2011).

When the auto covariance features of each model are taken into consideration, greater light is shown on the relationship that exists between the hierarchical FAR model and the DLM. Recalling $\mu_t(\tau) = Y_t(\tau) - \mu(\tau)$, let $C_\ell(\tau_1, \tau_2) = \mathbb{E}[\mu_t(\tau_1)\mu_{t-\ell}(\tau_2)]$ be the lag-` autocovariance function of {Yt}, which is time-invariant under stationary of {Yt}. Under model and assuming stationarity of {Yt}, the lag-1 autocovariance function is equivalently $C_1(\tau_1, \tau_2) = \mathbb{E}[\mu_t(\tau_1)\mu_{t-1}(\tau_2)] = \mathbb{E}\left[\left\{\int \psi(\tau_1, u)\mu_{t-1}(u)\, du + \epsilon_t(\tau_1)\right\}\mu_{t-1}(\tau_2)\right] = \int \psi(\tau_1, u)C_0(u, \tau_2)\, du$. For $\ell \geq 1$, we have the more general recursion $C_\ell(\tau_1, \tau_2) = \int \psi(\tau_1, u)C_{\ell-1}(u, \tau_2)\, du$, from which it is clear that each C is completely determined by the pair Now let $\boldsymbol{C}_\ell = \mathbb{E}[\boldsymbol{\mu}_t\boldsymbol{\mu}_{t-\ell}']$ be the lag-` auto covariance matrix for the vector valued time series {µt} in Equation. Under stationarity of $\{\boldsymbol{\mu}_t\}$, the lag-1 auto covariance matrix of $\boldsymbol{\mu}_t$ is $\boldsymbol{C}_1 = \mathbb{E}[\boldsymbol{\mu}_t\boldsymbol{\mu}_{t-1}'] = \mathbb{E}[\{\boldsymbol{\Psi}\boldsymbol{Q}\boldsymbol{\mu}_{t-1} + \boldsymbol{\epsilon}_t\}\boldsymbol{\mu}_{t-1}'] = \boldsymbol{\Psi}\boldsymbol{Q}\boldsymbol{C}_0$. notably, the relationship $\boldsymbol{C}_1 = \boldsymbol{\Psi}\boldsymbol{Q}\boldsymbol{C}_0$ is an approximation to the continuous version,

$C_1(\tau_1, \tau_2) = \int \psi(\tau_1, u) C_0(u, \tau_2)\, du,$ using the same quadrature approximation as in. More generally, the matrix recursion $C_\ell = \Psi Q C_{\ell-1}$ is a quadrature-based approximation to the continuous recursion, $C_\ell(\tau_1, \tau_2) = \int \psi(\tau_1, u) C_{\ell-1}(u, \tau_2)\, du$ for $\ell \geq 1$. Therefore, the evolution matrix ΨQ in the DLM induces a discrete approximation to the autocovariance structure in the hierarchical FAR model.

Conclusions

The evolution equation of resembles a VAR(1) on $\boldsymbol{\mu}_t = (\mu_t(\tau_1), \ldots, \mu_t(\tau_M))',$ but diverges significantly from a conventional VAR on YouTube in a number of significant respects. To begin, a well-defined fit of a VAR to yt can only be achieved if both the dimension mt and the observation points Tt remain constant across time. Only then can the fit be considered accurate. If this is not the case, then imputation is something that must be done. The conditional mean function and the conditional covariance function of the relevant Gaussian process are used in our technique, which allows for automated and efficient imputation of missing data. Second, since the observations are functional in character, it is quite probable that the components of yt are significantly connected with one another. The presence of significant collinearity in VARs may lead to overfitting, which in turn can have a negative impact on predicting and inference. Because the kernel function in our model is regularised with the use of a smoothness prior, we are able to reduce the negative impact that collinearity has on the estimate of. The smoothness prior on is an unconventional method of regularisation for VARs, but it is suitable for use in this environment. In the end, the quadrature matrix, Q, is included into the VAR coefficient matrix, Q, which reweights the vector t1 based on the information obtained from the evaluation points Te. This reweighting takes into account not just the vector values represented by t, but also the fact that the components of t correspond to ordered elements of Te, which do not necessarily have to be evenly spread out throughout the vector. The results of the simulations in Section reveal that our method offers significant gains in terms of its ability to predict in comparison to a VAR on yt.

The essential conclusions for vector-valued DLMs are expanded upon in the proof of Theorem , which may be found in the Appendix. The best linear predictors of Theorem minimise the risk R(, d) = supTe Re(, d) among all linear estimators, where the sup is taken across all finite Te T. This is equal to saying that the best linear predictors minimise the risk R(, d) = supTe Re(, d). The forecasting distributions [yt+h |Dt,] and [t+h |Dt,] for h > 0 are the most helpful examples of [|Y,] in Theorem. Other helpful examples include smoothing distributions [t |DT,] and filtering distributions [t |Dt,] for t = 1,..., T. The validity of Theorem is contingent on the observation points To under the only assumption that Zt is already established. In a broad sense, we will use the assumption that To Te, which means that Zt is an incidence matrix and is thus known. The proof of Theorem does not depend on To being arbitrarily dense in T, hence it may be used to designs that are either sparse or dense. For the purpose of implementation, we first calculate the appropriate expectations while the Gibbs sampling procedure is running (please see Appendix C), and then we take an average across the Gibbs sample of. Cressie and Wikle (2011) suggest that an EM method may be used instead to create an expectation estimate (Cressie and Wikle, 2011).

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