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A Brief Details of Cartesian plane and Its Development

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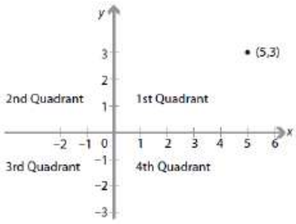
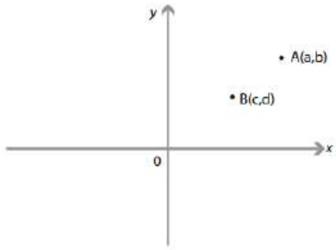
INTRODUCTION

The Cartesian plane

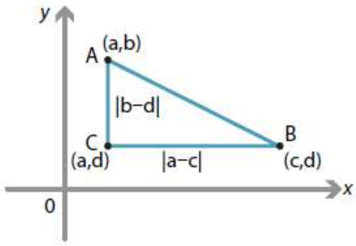
The numerical plane, also known as the Cartesian plane, is cut in half by two sharp lines known as the flat and the -bloc, sometimes known as the x bloc and the bloc. These axes intersect at a point that is referred to as the origin. Following the selection of a unit reserve, the location of each socket in the flat can be denoted by a figure pair that is arranged according to its position (,). For example, the -coordinate for the fact (5, 3) is 5, and the -coordinate for the same fact is 3, which are referred to as the first and second co-ordinates, respectively. When developing trigonometry, the four quadrants are typically referred to as the first, second, third, and fourth quadrants, as seen in this chart. This is because the first, second, and third quadrants are the most fundamental.





1.1 A váriety of basic questions concerning
a páir of A=(a,b) and B can be posed =(c,d).

The detachment between two facts unless the geometry distances overlap with one another, they are always positive. The degree of separation between A and B is same to that which exists between B and A. In order to get the formula for the detachment between two facts of the plane, we will first assume two positions: A(a, b), and B. (c,d). When fact C has coordinates, as shown in the graphic that accompanies this explanation, the angle formed by the triangle ABC at (a,d) must be right.

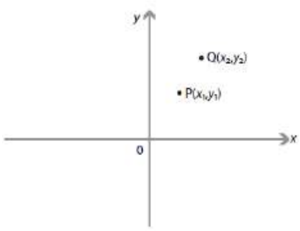


Now, with the help of Pythagoras formula, we may have
$AB^2 = |b - d|^2 + |a - c|^2 = (a - c)^2 + (b - d)^2$.

Evidentally $AB = \sqrt{(a-c)^2 + (b-d)^2}$.

The three-dimensional space is a similar formula as discussed in this section later.
The detachment formula
Suppose P=(η_1, λ_1) and Q=(η_2, λ_2) are 2- facts in the number-plane. Then
$PQ_2 = (\eta_2 - \eta_1)^2 + (\lambda_2 - \lambda_1)^2$
and so

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



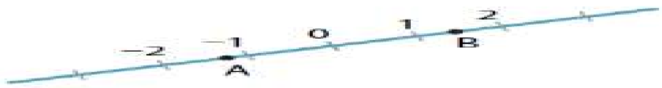
It is clear from the detachment formula that:

PQ = QP •

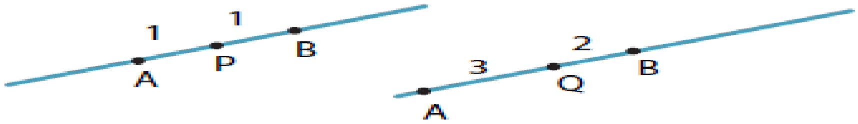
PQ = 0 if and only if P = Q.

Midfacts and division of an interval

Due to two facts A and B of the plane, it is apparent that a number line may be made on AB in order to label a (actual) number on each fact of AB.

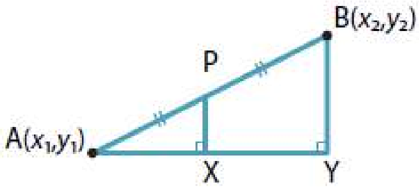


It can be (infinitely) done in various ways, but for geometric purposes it does not prove especially useful.On the other hand, we may state that in the following diagrams, fact P is the ratio of 1:1 between the AB interval, and fact Q is the ratio of 3:2 between the AB interval.



Midface of an interval

The midface of an interval AB is the fact that divides AB in the ratio 1 : 1.



Suppose fact A has coordinates (η1, λ1) and fact B contains coordinates (η2, λ2). It can be easily seen that the midfact P of AB is either congruent or similar:

$$\left(\frac{\eta_1 + \eta_2}{2}, \frac{\lambda_1 + \lambda_2}{2}\right)$$

Coordinate Geometry is considered to be one of the most interesting concepts of mathematics. Coordinate Geometry (or the analytic geometry) describes the link between geometry and algebra through graphs involving curves and lines. It provides geometric aspects in Algebra and enables them to solve geometric problems. It is a part of geometry where the position of points on the plane is described using an ordered pair of numbers. Here, the concepts of coordinate geometry (also known as Cartesian geometry) are explained along with its formulas and their derivations.

Introduction to Coordinate Geometry

Coordinate geometry (or analytic geometry) is defined as the study of geometry using the coordinate points. Using coordinate geometry, it is possible to find the distance between two points, dividing lines in m:n ratio, finding the mid-point of a line, calculating the area of a triangle in the Cartesian plane, etc. There are certain terms in Cartesian geometry that should be properly understood. These terms include:

What is a Co-ordinate and a Co-ordinate Plane?

You must be familiar with plotting graphs on a plane, from the tables of numbers for both linear and non-linear equations. The number line which is also known as a Cartesian plane is divided into four quadrants by two axes perpendicular to each other, labelled as the x-axis (horizontal line) and the y-axis(vertical line).

The four quadrants along with their respective values are represented in the graph below-

- Quadrant 1 : (+x, +y)
- Quadrant 2 : (-x, +y)
- Quadrant 3 : (-x, -y)
- Quadrant 4 : (+x, -y)

The point at which the axes intersect is known as the **origin**. The location of any point on a plane is expressed by a pair of values (x, y) and these pairs are known as the **coordinates**.
The figure below shows the Cartesian plane with coordinates (4,2). If the coordinates are identified, the distance between the two points and the interval's midpoint that is connecting the points can be computed.

Equation of a Line in Cartesian Plane

Equation of a line can be represented in many ways, few of which is given below-

Coordinate Geometry Terms	
Coordinate Geometry Definition	It is one of the branches of geometry where the position of a point is defined using coordinates.
What are the Coordinates?	Coordinates are a set of values which helps to show the exact position of a point in the coordinate plane.
Coordinate Plane Meaning	A coordinate plane is a 2D plane which is formed by the intersection of two perpendicular lines known as the x-axis and y-axis.
Distance Formula	It is used to find the distance between two points situated in $A(x_1,y_1)$ and $B(x_2,y_2)$
Section Formula	It is used to divide any line into two parts, in m:n ratio
Mid-Point Theorem	This formula is used to find the coordinates at which a line is divided into two equal halves.

(i) General Form

The general form of a line is given as Ax + By + C = 0.

(ii) Slope intercept Form

Let x, y be the coordinate of a point through which a line passes, m be the slope of a line, and c be the y-intercept, then the equation of a line is given by:

y=mx + c

(iii) Intercept Form of a Line

Consider a and b be the x-intercept and y-intercept respectively, of a line, then the equation of a line is represented as-

y = mx + c

Slope of a Line:

Consider the general form of a line Ax + By + C = 0, the slope can be found by converting this form to the slope-intercept form.

Ax + By + C = 0

⇒ By = − Ax – C

or,

$$\Rightarrow y = -\frac{A}{B}x - \frac{C}{B} \quad y = -\frac{A}{B}x - \frac{C}{B} \quad y = -\frac{A}{B}x - \frac{C}{B}$$

Comparing the above equation with y = mx + c,

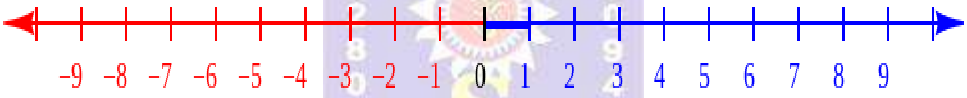
$$m = -\frac{A}{B}$$

Thus, we can directly find the slope of a line from the general equation of a line.
Coordinate Geometry Formulas and Theorems
Distance Formula: To Calculate Distance Between Two Points
Let the two points be A and B, having coordinates to be (x_1, y_1) and (x_2, y_2), respectively.
Thus, the distance between two points is given as- Midpoint Theorem: To Find Mid-point of a Line Connecting Two Points

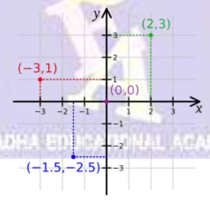
Consider the same points A and B, which have coordinates (x_1, y_1) and (x_2, y_2), respectively. Let M(x,y) be the midpoint of lying on the line connecting these two points A and B. The coordinates of point M is given as- Angle Formula: To Find The Angle Between Two Lines
Consider two lines A and B, having their slopes m_1 and m_2, respectively.
Let **"θ"** be the angle between these two lines, then the angle between them can be represented as-
Section Formula: To Find a Point Which Divides a Line into m:n Ratio
Consider a line A and B having coordinates (x_1, y_1) and (x_2, y_2), respectively. Let P be a point that which divides the line in the ratio m:n, then the coordinates of the coordinates of the point P is given as-
Students can follow the link provided to learn more about the section formula along its proof and solved examples.
Area of a Triangle in Cartesian Plane
The area of a triangle In coordinate geometrywhose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is
If the area of a triangle whose vertices are (x_1, y_1),(x_2, y_2) and (x_3, y_3) is zero, then the three points are collinear.
Common coordinate systems

Number line

The simplest example of a coordinate system is the identification of points on a line with real numbers using the number line. In this system, an arbitrary point O (the origin) is chosen on a given line. The coordinate of a point P is defined as the signed distance from O to P, where the signed distance is the distance taken as positive or negative depending on which side of the line P lies. Each point is given a unique coordinate and each real number is the coordinate of a unique point

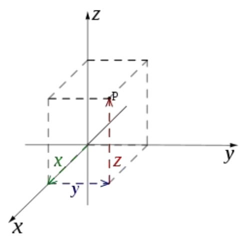


Cartesian coordinate system



The Cartesian coordinate system in the plane.

The prototypical example of a coordinate system is the Cartesian coordinate system. In the plane, two perpendicular lines are chosen and the coordinates of a point are taken to be the signed distances to the lines.



In three dimensions, three mutually orthogonal planes are chosen and the three coordinates of a point are the signed distances to each of the planes. This can be generalized to create n coordinates for any point in n-dimensional Euclidean space.
Depending on the direction and order of the coordinate axes, the three-dimensional system may be a right-handed or a left-handed system. This is one of many coordinate systems.

Some Problems Of Coordinate Geometry

A triangle has vertices A (0, 2), B (8, 2) and C (4, 6).
(a) Find the lengths of AB, BC, and AC.
(b) Show that the triangle ABC is an isosceles triangle.

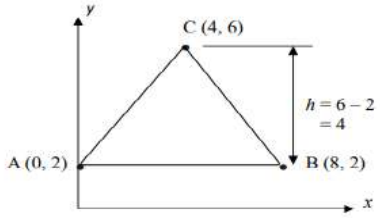
(c) Find the area of triangle ABC

$$\text{length of } BC = \sqrt{(6-2)^2 + (4-8)^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32}$$
$$= 4\sqrt{2} \text{ units} \quad (ans)$$

$$\text{length of } AC = \sqrt{(6-2)^2 + (4-0)^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32}$$
$$= 4\sqrt{2} \text{ units} \quad (ans)$$

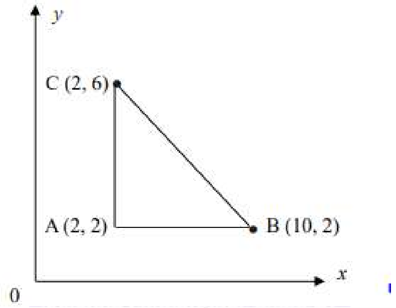
(b) Since *BC* and *AC* have identical lengths, *ABC* is an isosceles triangle. (ans)

(c)



$$\text{The area of triangle ABC} = \frac{1}{2} \times 8 \times 4$$

The coordinates of triangle ABC are A (2, 2), B (10, 2), C (2, 6).
(a) Find the lengths of AB, BC, and AC.
(b) Show that the triangle ABC is a right–angled triangle.
(c) Find the perpendicular distance from A to the line BC



$$a) \quad \text{length of } AB = \sqrt{(10-2)^2 + (2-2)^2}$$
$$= \sqrt{64}$$
$$= 8 \text{ units} \quad (ans)$$

$$\text{length of } BC = \sqrt{(2-10)^2 + (6-2)^2}$$
$$= \sqrt{64+16}$$
$$= \sqrt{80}$$
$$= 4\sqrt{5} \text{ units} \quad (ans)$$

$$\text{length of } AC = \sqrt{(2-2)^2 + (6-2)^2}$$
$$= \sqrt{16}$$
$$= 4 \text{ units} \quad (ans)$$

$$b) \quad BC^2 = \left(\sqrt{80}\right)^2 = 80$$
$$AB^2 = (8)^2 = 64$$
$$AC^2 = (4)^2 = 16$$

Since

Since $BC^2 = AB^2 + AC^2$, by Pythagoras Theorem, ABC is a right-angled triangle. (ans)

c) Let h = perpendicular distance from A to the line BC

Area of triangle ABC $= \frac{1}{2} \times 8 \times 4$

$= 16\ units^2$

$$16 = \frac{1}{2} \times 4\sqrt{5} \times h$$

$$h = 3.58\ unit\quad (ans)$$

Cartesian coordinate system

One of the most well-known types of coordinate system is known as the Cartesian coordinate system. In the plane, two lines that are perpendicular to one another are picked, and the signed distances that separate them are used to get the coordinates of a point. A point in three dimensions is represented by its three coordinates, which are the signed distances between three different planes that are mutually orthogonal with one another. Using this approach, the n coordinates of every point in an n-dimensional space in Euclidean space may be determined. Depending on the direction of the coordinate axes and the sequence in which they are presented, the three-dimensional system can either be right-handed or left-handed. This is only one of the many available coordinate systems; there are many more.

COORDINATES SYSTEM

In analytic geometry, a coordinate system is assigned to the plane, and as a result, every point in the plane is assigned a pair of real number coordinates. In a similar manner, coordinates are assigned to Euclidean space, and each point in this space has three coordinates. The significance of the coordinates is determined by the starting point that is selected as the origin. There are many other kinds of coordinate systems, but the most typical ones are the Cartesian coordinates, which are shown here (in a plane or space)

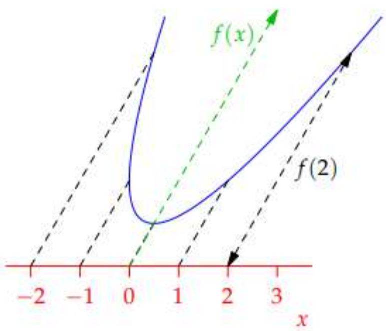
The Cartesian coordinate system is by far the most widely used coordinate system. In this system, each point has an x-coordinate that represents its location on the horizontal plane, and a y-coordinate that represents its position on the vertical plane. These are almost often presented in the form of an ordered pair (x, y). This method may also be used for the study of three-dimensional geometry, in which each point in Euclidean space is characterized by a well-organized set of three coordinates (x, y, z).

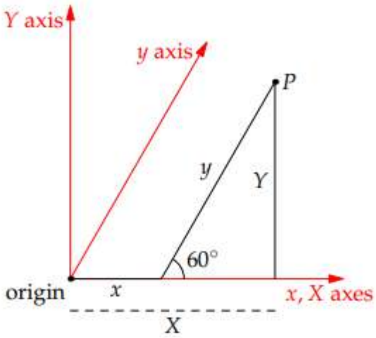
Calculating Polar Coordinates (in a plane)

Every point on the plane is given a representation in polar coordinates by the distance r that it is from the origin and the angle that it makes with the positive x-axis. The angle is typically measured in the counterclockwise direction. In this notation, points are almost always represented by an ordered pair, which looks like this: (r,). The following formulas may be used to perform the transformation between Cartesian coordinates and polar coordinates in a two-dimensional space:

This system may be generalized to three-dimensional space through the use of cylindrical or spherical coordinates.

Here is an alternative description of a parabola. This time the function is $f(x) = x^2 + 1$. Notice the slope: with only one axis, Descartes and Fermat could measure the distance to the curve using parallels inclined at whatever angle they liked. In a modern sense, this example has a second axis, drawn in green, inclined 30° to the vertical.





If this makes you nervous, you can perform a change of basis calculation from linear algebra: the point P in the second picture has co-ordinates (X,Y) relative to 'usual' orthogonal Cartesian axes; its co-ordinates are (x, y) relative to the slanted axes. It is easy to see that

$$\begin{cases} X = x + y\cos 60° = x + \frac{1}{2}y \\ Y = y\sin 60° = \frac{\sqrt{3}}{2}y \end{cases}$$

For any point on the curve, we then have

$$\sqrt{3}X - Y = \sqrt{3}x \implies (\sqrt{3}X - Y)^2 = 3x^2 = 3(y-1) = 3\left(\frac{2}{\sqrt{3}}Y - 1\right)$$
$$\implies 3X^2 - 2\sqrt{3}XY + Y^2 - 2\sqrt{3}Y + 3 = 0$$

which recovers the implicit equation for the parabola relative to the standard orthogonal axes.5 Other curves could be similarly described. Descartes was comfortable with curves having implicit equations. The standardized use of a second axis orthogonal to the first was instituted in 1649 by Frans van Schooten; this immediately gives us the modern notion of the co-ordinates.

Descartes used his method to solve several problems that had proved much more difficult synthetically such as finding complicated intersections. geometric proofs of all assertions to back up his algebraic work (similarly to how Islamic mathematicians had proceeded). He was not shy about his discovery however, stating that, once several examples were done, it wasn't necessary to draw physical lines and provide a geometric argument, the algebra was the proof. This point of view was controversial at the time, but over the following centuries it eventually won out.

As an example of the power of analytic geometry, consider the following result.

Theorem. The medians of a triangle meet at a common point (the centroid), which lies a third of the way along each median. This can be done using pure Euclidean geometry, though it is somewhat involved.

It is comparatively easy in analytic geometry.

Proof. Choose axes pointing along two sides of the triangle with with the origin as one vertex.6

If the side lengths are a and b, then the third side has equation $bx + ay = ab$ or $y = b - \frac{b}{a}x$.

The midpoints now have co-ordinates:

$$\left(\frac{a}{2}, 0\right), \left(0, \frac{b}{2}\right), \left(\frac{a}{2}, \frac{b}{2}\right)$$

Now compute the point 1/3 of the way along each median: for instance

$$\frac{2}{3}\left(\frac{a}{2}, 0\right) + \frac{1}{3}(0, b) = \frac{1}{3}(a, b)$$

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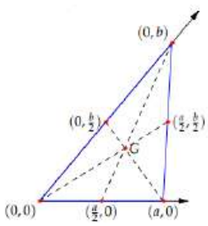
$$\frac{2}{3}\left(\frac{a}{2}, 0\right) + \frac{1}{3}(0, b) = \frac{1}{3}(a, b)$$

One obtains the same result with the other medians.

e

With the assistance of his notation, Descartes made many other mathematical breakthroughs. For instance, he was able to state a critical part of the Fundamental Theorem of Algebra, the factor theorem: if a is a root of a polynomial, then x − a is a factor. He didn't give a complete proof of this fact as he thought it to be self-evident, perhaps because his notation made it so easy to work with polynomials. The full theorem7 wasn't proved until 1821 (by Cauchy). The factor theorem is essentially a corollary of the division algorithm for polynomials: if f(x), g(x) are polynomials, then there exist unique polynomials q(x),r(x) for which



$$f(x) = q(x)g(x) + r(x) \qquad \deg r < \deg g$$

If deg g = 1, then r is necessarily constant. Suppose $g(x) = x - a$ Then $r = 0$.

The Beginnings of Calculus

- At the heart of calculus is the relationship between velocity, displacement, rate of change and area.
  - The instantaneous velocity of a particle is the rate of change of its displacement.
  - The displacement of a particle is the net area under its velocity-time graph.

To state such principles essentially requires graphs and some form of analytic geometry (rate of change means slope. . .). Once these appeared in the early 1600's, the rapid development of calculus was arguably inevitable. However, many of the basic ideas were in place prior to Descartes and Fermat. In the context of the above, the Fundamental Theorem of Calculus intuitively states that complete knowledge of displacement is equivalent to complete knowledge of velocity. Of course, the modern statement is far more daunting:

Example 1. Consider the triangle defined by corners (1, 0, 0), (1, 0, −1), and (1, −1, 0), which represent the three vertices a, b, and c, respectively. Let u = 0.2 and v = 0.4, where 0 < u + v < 1. Then, based on Equation (1), the coordinate triplet of point p is (1, −0.2, −0.4). If u = 0.8 and v = 0.4 (0 < u + v < 2), then the coordinate triplet of point q is (1, −0.8, −0.4) (see Figure 3).

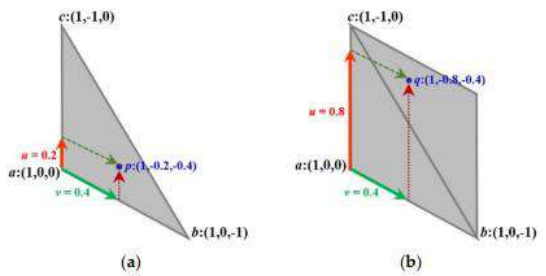


Figure 3. A composition of the barycentric technique and discrete coordinate system to address points p and q in the triangular plane by coordinate triplets in (a) and (b), respectively.

4.2 Continuous Coordinate System for Reflecting the Triangular Symmetry

In order to construct a continuous triangular coordinate system that is reliable, we combine the discrete coordinate system that is used for the triangle grid with the BCS. Because of this, we are able to create a triangular coordinate system that may be used for an infinite amount of time. Integer coordinate triplets with a range of various sums were used in the discrete triangular coordinate system. The coordinate triplets that relate to the points that are included inside a triangle have BCS values that are expressed as fractions. We come up with an entirely new approach that takes use of triplets over the whole of the plane. We start by subdividing each equilateral triangle that makes up the triangular grid into three isosceles obtuse-angled triangles, as shown in Figure 4. This is the first step in the process. The regions of these triangles will be denoted by the letters A, B, and C, respectively. In this particular situation, the beginning point, which is marked by the letter an in Equation, will be the location that is precisely midway between the two zones, which is represented by the letter m in the equation (1). This position will serve as the beginning point for the calculation of the coordinates of the points that are situated in the three areas that are designated A, B, and C respectively.

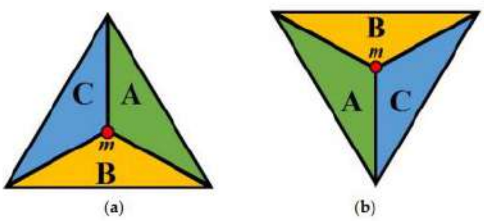


Figure 4. Dividing positive (**a**) and negative (**b**) triangles to three areas A, B, and C. The letters assigned to the isosceles triangles are based on the orientation of the sides.

As was just discussed above, in order to determine the coordinates of these midpoints, we employ coordinate triplets with sums of either +1 or -1, depending on the orientation of the original triangle. The point in the middle of the positive triangles is represented by the sum of 1, while the point in the middle of the negative triangles is represented by the total of -1. Therefore, by using the barycentric Equation (1) based on these midpoints, we are able to generate a unique triplet for each point located in each section of the plane, which will be described further down in this section.

Because the values of u and v are constrained by the range 0 u + v 1 (whether inside or on the border of the given triangle), we are able to address the points that fall within the areas A, B, and C of each type of triangle (,) independently. This is because the barycentric equation (1) tells us that the values of u and v are limited in this way. Let us, however, examine the scenario in which the sum of u and v meets the requirement that 0 u + v 2, so that the criteria 0 u 1 and 0 v 1 remain true. Then, as a consequence, each midpoint may be used to address not just the points in the region contained in this initial triangle, but also the points in the nearby area that are marked with the same letters. This is because the letters that signify these points are the same. In order to demonstrate this point, the green region shown in Figure 5a may be entirely addressed by using either the midpoint a(+) or a() as the beginning point in Equation (1).

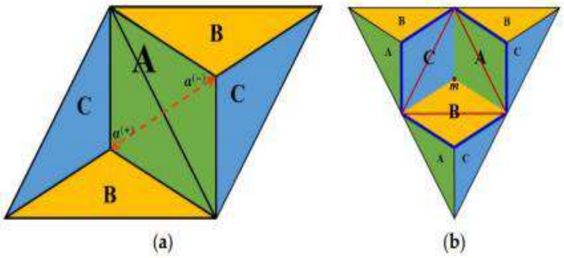


Figure 5. (**a**) By using either $a^{(+)}$ or $a^{(-)}$, the whole green area A could be addressed. (**b**) The hexagon surrounded by the thick dark blue line shows the entire area that can be addressed by using a positive midpoint m.

CONCLUSION

1. In the generalised coordinate system, each auxiliary coordinate system must include a total of six coordinates, including three linear coordinates and three angular coordinates for each system.
2. In order to limit the amount of information that can be gleaned from a datum, the collection of datums may include both linear and angular coordinates inside the system. However, the development of the coordinate system does not make use of these particular coordinates in any way.
3. As a general rule, the tolerances of linear and angular coordinates are symmetrical, and one ought to make use of tolerance systems for linear and angular dimensions of size features for the purpose of tolerancing them.
4. This brings us to our final point. The application of a coordinate system for the purpose of standardising position and orientation changes demonstrates conformity with a systematic approach, which ultimately leads to an increase in product quality with regard to the appropriateness of geometric requirements. It is very necessary for the coordinate system to be included in the GPS geometry matrix model when the starting position is being modelled. In order to standardise all of the geometric specifications of products, including but not limited to coordinates, dimensions of size characteristics, and deviations in the form of surfaces, a common coordinate system is required. This is the case because of the purpose of standardising all of the geometric specifications of products.

SUGGESSIONS

In this paper, we detailed the ongoing research that we are conducting on the use and formalisation of algebraic approaches for the theorem proving in geometry. In this project, the formalisation and implementation of algebraic techniques are the primary objectives. It is envisaged that the research would make it easier for educational institutions to make wider use of algebraic methods and will contribute to the formalisation of mathematical concepts.

1. In the generalised coordinate system, each auxiliary coordinate system must include a total of six coordinates, including three linear coordinates and three angular coordinates for each system.
2. In order to limit the amount of information that can be gleaned from a datum, the collection of datums may include both linear and angular coordinates inside the system.

However, the development of the coordinate system does not make use of these particular coordinates in any way.

3. As a general rule, the tolerances of linear and angular coordinates are symmetrical, and one ought to make use of tolerance systems for linear and angular dimensions of size features for the purpose of tolerancing them.

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