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Multi-Resolution Analysis and Wavelet Collocation Techniques for Modeling Viscous Fluid Dynamics in Irregular Geometries

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Abstract

In order to numerically solve the incompressible Navier-Stokes equations controlling the flow of viscous fluids in domains with irregular, curved, or changing borders, this research builds a multi-resolution, wavelet-based collocation technique. Utilizing the principles of multiresolution analysis (MRA) and interpolating (Deslauriers-Dubuc) wavelets, we develop an adaptive point representation. This representation automatically adjusts the local density of collocation points in areas with a high solution gradient, such as boundary layers, shear layers, and vertical structures. In contrast, it stays coarse in smooth regions, allowing us to focus computational resources where they are most needed. A volume-penalization (Brinkman) formulation adds complex and irregular geometries to the Cartesian wavelet grid in a way that prevents body-fitted meshing while maintaining the wavelet basis's compression and differentiation features. For the velocity-pressure system, the resultant adaptive wavelet collocation method (AWCM) is connected to a semi-implicit time-marching methodology. We derive theoretical error estimates from the relationship between wavelet threshold, vanishing-moment order, and solution regularity. To demonstrate the method's expected compression rates and convergence behavior, we use two representative benchmark configurations: lid-driven cavity flow with a re-entrant corner and flow past a circular obstacle in a channel. Wavelet collocation provides a computationally efficient and theoretically sound substitute for classical body-fitted discretization in multiscale viscous flow problems, according to the results.

Keywords: wavelet collocation; multiresolution analysis; incompressible Navier–Stokes equations

1. Introduction

Among the many uses of computational fluid dynamics (CFD), from biomedical flow in vascular geometries to aerodynamic flow around complicated structures, is the numerical simulation of viscous incompressible flow in domains bounded by irregular or curved surfaces. A body-fitted mesh that adheres to the domain boundary is usually required for classical discretization strategies, such as finite difference, finite volume, and finite element approaches. It is computationally costly to generate and maintain such meshes for irregular or moving geometries, and while the solution has localized features like boundary layers, wakes, or thin shear layers that occupy only a small fraction of the computational domain, uniform or quasi-uniform meshes are inefficient.

An alternate that is intrinsically well-suited to such multiscale issues is the wavelet-based approach. A function can be sparsely represented using a wavelet basis, which allows for simultaneous localization in physical space and scale (frequency). This means that a small number of fine-scale wavelets concentrated where they are needed can capture sharp local features, while a few large-scale coefficients are sufficient for smooth regions. Wavelets are an ideal foundation for adaptive discretization of PDEs due to this property and the vanishing-moment constraint, which suppresses wavelet coefficients in areas of local polynomial smoothness.

This paper presents an adaptive wavelet collocation method (AWCM) for the incompressible Navier-Stokes equations, focusing on two important features for real-world applications in non-standard geometries: (i) building the adaptive, evolving computational grid from the multiresolution structure of interpolating wavelets, and (ii) enforcing no-slip and other boundary conditions on non-Cartesian boundaries without body-fitted meshing, accomplished

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by incorporating a volume-penalization formulation into the wavelet framework in a consistent and efficient manner.

What follows is an outline of the rest of the paper. The second section provides a literature overview on PDE wavelet approaches and multiresolution analysis. In Section 3, we provide the groundwork mathematically for multiresolution analysis and interpolating wavelets and present the governing equations. Learn about the Navier-Stokes system's adaptive wavelet collocation discretization in Section 4. Section 5 details the volume penalization treatment of irregular geometries. Here is a brief overview of the whole numerical algorithm in Section 6. Section 8 gives the related convergence and error analysis, while Section 7 talks about typical benchmark configurations. Sections 9 and 10 wrap up the paper and go over the findings.

2. Literature Review

To solve PDEs on spherical domains, *Mehra and Kevlahan (2008)* devised an adaptive wavelet collocation technique. By adjusting computational resolution to areas requiring greater accuracy, the study hoped to show how wavelet-based multiresolution approaches may efficiently describe localized structures. The approach made use of dynamic refinement and adaptive wavelet collocation. Wavelets were able to reduce needless computations in smooth regions and manage complex geometrical domains, according to the results. Researchers found that adaptive wavelet approaches work well for non-Cartesian and irregular geometries PDE modeling. For the extended Burgers-Huxley equation, a significant nonlinear evolution equation associated with convection and diffusion processes, *Shiralashetti and Kumbinarasaiah (2018)* devised a Cardinal B-spline wavelet numerical approach. Our goal was to use localized wavelet approximations to get precise numerical solutions. Transformed the governing equation into an algebraic system by means of truncated B-spline wavelet expansions. The numerical results were in excellent agreement with the answers used as references. In order to resolve nonlinear transport processes with limited spatial features, the authors found that wavelet bases work well.

Working with coupled nonlinear differential equations that arise in rotating micropolar nanofluid flow, *Kumbinarasaiah, Raghunatha and co-authors (2022)* utilized the Hermite wavelet technique. The goal was to examine the impact of nanoparticles, microstructure, and rotation on a complicated fluid system. By using Hermite wavelet approximations, the approach converted the coupled nonlinear equations into an algebraic system. The numerical findings showed that the answers for various physical parameters were steady and accurate. Based on the results, wavelet approaches are a good fit for nonlinear fluid-flow models with strongly coupled parameters.

For fractionally coupled Navier-Stokes equations in multiple dimensions, *Kumbinarasaiah (2022)* created a Hermite wavelet method. In this research, we used wavelet collocation to try to solve nonlinear multidimensional fluid equations numerically. We theoretically investigated convergence after transforming the fractional Navier-Stokes equations into nonlinear algebraic systems using Hermite wavelet approximations at chosen collocation locations. The results proved that the procedure was accurate and applicable. For problems of the multidimensional Navier-Stokes type, the study found that wavelet collocation works quite well.

Using a wavelet approach, *Kumbinarasaiah and Raghunatha (2022)* tackled the classic Jeffery-Hamel flow issue. This work aimed to find numerical solutions to the problem of viscous flow in non-uniform flow domains, specifically in the context of converging or diverging channels. We used a wavelet-based approximation technique to handle the governing nonlinear differential equation. The outcomes faithfully depicted shifts in velocity behavior as a function of different physical parameters. The authors came to the conclusion that wavelet approaches are a powerful tool for studying flows of viscous fluids in channels with geometrical variations.

For multi-homogeneous boundary-layer flow of a viscous electrically conducting fluid over a stretching plate, *Kumbinarasaiah and Preetham (2023)* presented the Bernoulli Wavelet

Collocation Method. The goal was to find a solution to the flow-governing nonlinear Falkner-Skan-type problem. The collocation framework made use of a newly constructed Bernoulli wavelet operational matrix. When applied to certain parameters, the approach was able to capture changes in the boundary layer as well as overshoots, undershoots, and dual solutions. According to the research, nonlinear viscous boundary-layer problems can be accurately and flexibley solved using Bernoulli wavelet collocation.

In *2023, Preetham, Kumbinarasaiah, and Raghunatha* used the Hermite wavelet method to study the compressive flow of an electrically conducting Casson fluid. Examining nonlinear flow behavior in the presence of rheological and magnetic influences was the primary goal. We numerically solved the altered coupled equations using the Hermite wavelet approximation. The findings correctly depicted variations brought about by fluid and magnetic factors and proved the existence of stable solutions. The research found that complex squeezing-flow topologies with non-Newtonian viscous fluids are well-suited to wavelet-based methods. For the squeeze-film lubrication problem of porous journal bearings with couple-stress fluid, *Shiralashetti and Badiger (2023)* devised an algebraic multigrid technique using Haar wavelets. The goal was to offer a multiresolution numerical solution for lubrication flow in constrained curved geometries that is efficient. We solved the governing equations using a combination of algebraic multigrid and Haar wavelets. When applied numerically, the procedure yielded accurate results for lubrication and pressure properties. According to the authors, computational fluid dynamics in complicated bearing geometries can be better handled using multiresolution wavelet approaches.

Applying a wavelet lifting method, *Shiralashetti and Badiger (2023*) tackled a micropolar lubrication problem involving a porous journal bearing. This research set out to simplify computer analysis of non-classical lubricating flows. The technique built an effective numerical model of the governing equations using wavelet lifting. The approach successfully captured the impacts of micropolar properties and porosity on lubrication behavior, according to the results. Researchers found that fluid-flow models with non-standard engineering geometries and those with curves benefit from wavelet lifting techniques. In their study, *Shiralashetti, Harishkumar, and Hanaji (2023)* utilized a Legendre wavelet operational matrix approach to analyze natural convection flowing over a vertical plate within a saturated porous material that experiences thermal radiation. Solving coupled momentum and heat-transfer equations numerically with high accuracy was the goal. We reduced the nonlinear differential system using Legendre wavelet operational matrices. The findings accurately depicted the impact of radiation and characteristics of the porous media on the distributions of flow and temperature. The authors found that coupled viscous-flow and heat-transfer models are best implemented using wavelet operational matrices.

In order to study the unsteady multi-scale Hamiltonian flow of a nanofluid across a porous media, *Preetham and Kumbinarasaiah (2024)* used Fibonacci wavelets. Nonlinear fluid and thermal behavior under magnetic, porous, viscous, and time-dependent influences was the intended investigation. The altered governing equations were numerically approximated using a Fibonacci wavelet series collocation method. The outcomes accurately depicted how velocity and thermal properties changed with varying parameter settings. The research found that sophisticated wavelet collocation methods may solve complicated non-Newtonian fluid-flow problems in a computationally efficient and accurate manner.

3. Mathematical Preliminaries

3.1 Governing Equations

The motion of a viscous, incompressible Newtonian fluid occupying a possibly irregular open domain $\Omega \subset \mathbb{R}^d$ ($d = 2, 3$) with boundary $\partial\Omega$ is governed by the incompressible Navier–Stokes equations

$$\partial u/\partial t + (u\cdot\nabla)u = -\nabla p/\rho + \nu\nabla^2 u + f, \qquad \nabla\cdot u = 0, \qquad x \in \Omega,\ t > 0,$$

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subject to the no-slip condition $u = 0$ on $\partial\Omega$ (or a prescribed velocity for moving boundaries) and an initial condition $u(x,0) = u_0(x)$. Here $u(x,t)$ denotes the velocity field, $p(x,t)$ the pressure, ρ the (constant) density, ν the kinematic viscosity, and f a body-force term. Equation set above will be discretized directly on the adaptive point set generated by the wavelet transform described in Section 4, rather than on a body-fitted mesh.

3.2 Multiresolution Analysis

A multiresolution analysis (MRA) of $L^2(\mathbb{R})$ is a nested sequence of closed subspaces

$$\ldots \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \subset \ldots \subset L^2(\mathbb{R}), \quad \text{with} \;\; \bigcap_j V_j = \{0\}, \;\; \bigcup_j V_j \text{ dense in } L^2(\mathbb{R}),$$

such that $f(x) \in V_j$ if and only if $f(2x) \in V_{j+1}$, and V_0 is generated by integer translates of a single scaling function φ, i.e. $V_j = \mathrm{span}\{\varphi_{j,k}(x) = 2^{(j/2)}\varphi(2^j x - k) : k \in \mathbb{Z}\}$. The scaling function satisfies a two-scale refinement equation

$$\varphi(x) = \Sigma_k\, h_k\, \varphi(2x - k),$$

for a sequence of filter coefficients $\{h_k\}$. The orthogonal complement W_j of V_j in V_{j+1} is spanned by dilations and translations of a wavelet function ψ, so that any function $f \in L^2(\mathbb{R})$ admits the multiresolution expansion

$$f(x) = \Sigma_k\, c_{j0,k}\, \varphi_{j0,k}(x) + \Sigma_{j=j0}^{\infty} \Sigma_k\, d_{j,k}\, \psi_{j,k}(x),$$

where the scaling coefficients $c_{j,k}$ capture the coarse-scale approximation of f at level j and the wavelet coefficients $d_{j,k}$ capture the detail added in passing from level j to level $j + 1$. If ψ has M vanishing moments ($\int x^m\psi(x)dx = 0$ for $m = 0,\ldots,M - 1$), then $d_{j,k}$ decays rapidly wherever f is locally smooth, and remains large only near genuine local features such as boundary layers or fronts; this decay property is the basis for the adaptive grid construction in Section 4.

3.3 Interpolating Wavelets

Collocation methods require a basis whose scaling function has the cardinal interpolation property $\varphi(k) = \delta_{0,k}$. Following Deslauriers and Dubuc, an interpolating scaling function of polynomial exactness order N is constructed via Lagrange interpolation on dyadic point sets, and the associated interpolating wavelet is defined through the same two-scale relation. A key computational advantage of this construction is that the scaling coefficients coincide with point values of the function being represented, $c_{j,k} = f(x_{j,k})$, so that no separate quadrature or projection step is required to move between physical space and wavelet space: the fast wavelet transform (FWT) acts directly on nodal values through a lifting-scheme (predict-and-update) algorithm with computational cost $O(N_{points})$.

4. Adaptive Wavelet Collocation Method

4.1 Sparse Point Representation

At each time step, the velocity field is represented on a dynamically adapted set of collocation points $X = \{x_{j,k}\}$, built by retaining, at every resolution level j from a coarse reference level j_0 up to a maximum level J, only those wavelet coefficients whose magnitude exceeds a prescribed threshold ε:

$$X = \{\, x_{j,k} : |d_{j,k}| \geq \varepsilon \cdot 2^{(-(J-j))}\,,\;\; j_0 \leq j \leq J \,\},$$

together with a fixed neighbourhood of adjacent points at each level required to maintain interpolation accuracy (the interpolation, or safety, zone). The threshold ε controls the trade-off between accuracy and grid size, and is chosen, via the analysis of Section 8, so that the resulting truncation error is commensurate with the time-discretization error.

4.2 Grid Adaptation Procedure

Because the flow field evolves in time, the collocation point set must itself evolve. At each time step the following operations are performed:

Forward wavelet transform of the current velocity field to obtain the coefficients $d_{j,k}$ at all active levels.

Thresholding: coefficients with $|d_{j,k}| < \varepsilon$ are discarded, defining the significant set.

Grid extension: points adjacent (in space and in the next-finer level) to significant points are added to anticipate the possible propagation of fine-scale features between time steps.

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Inverse wavelet transform to recover nodal velocity values on the new adaptive point set.
This predict-then-refine strategy guarantees that the adaptive grid can track emerging boundary layers or vortical structures without a full uniform-resolution computation at any stage.

4.3 Differentiation on the Adaptive Grid

Spatial derivatives required by the convective and viscous terms of the Navier–Stokes equations are evaluated directly on the wavelet interpolant. Because the interpolating scaling function is a piecewise polynomial of exactness order N, the wavelet interpolant

$$u\_J(x) = \Sigma_k\, c_{j0,k}\, \varphi_{j0,k}(x) + \Sigma_{j=j0}\hat{}(J-1)\, \Sigma_k\, d_{j,k}\, \psi_{j,k}(x)$$

can be differentiated term by term, since the derivatives of φ and ψ can themselves be pre-computed at the interpolation points using the refinement equation (connection-coefficient approach). This provides derivative approximations that are accurate to order N on smooth regions of the grid while remaining well defined at the irregular, non-uniformly spaced collocation points produced by adaptation.

4.4 Time Integration

The pressure–velocity coupling is treated with a projection (fractional-step) method: an intermediate velocity field is advanced explicitly (or with a low-storage Runge–Kutta scheme) using the convective and viscous terms evaluated on the current adaptive grid, after which a pressure-Poisson equation is solved to enforce the divergence-free constraint and correct the velocity field. The viscous term, which is stiff on fine wavelet levels, is treated implicitly (Crank–Nicolson) to remove the restrictive diffusive stability limit, while the pressure-Poisson problem is solved by a wavelet-adapted iterative (Krylov subspace) solver preconditioned by the coarse-level scaling-function operator.

5. Treatment of Irregular Geometries

5.1 Volume-Penalization (Brinkman) Formulation

Rather than constructing a body-fitted computational mesh, the irregular fluid domain Ω is embedded within a larger, simple computational box $\tilde{\Omega} \supset \Omega$ that is discretized by the Cartesian wavelet grid described in Section 4. The complement $\Omega^s = \tilde{\Omega} \setminus \Omega$, representing the solid or excluded region, is treated as a porous medium of vanishingly small permeability $\eta \ll 1$ through the addition of a penalization term to the momentum equation:

$$\partial u/\partial t + (u\cdot\nabla)u = -\nabla p/\rho + \nu\nabla^2 u + f - (1/\eta)\,\chi\Omega^s(x)\, u,$$

where $\chi\Omega^s$ is the characteristic (mask) function of the solid region. In the limit $\eta \to 0$, it can be shown that the solution of the penalized system converges, at a rate proportional to $\sqrt{\eta}$, to the solution of the original Navier–Stokes equations posed on Ω with the no-slip condition on ∂Ω. This formulation replaces the geometric complexity of mesh generation with a single smooth (or sharply varying) coefficient field $\chi\Omega^s/\eta$, which is naturally handled by the wavelet framework: the mask function introduces a thin transition layer of width $O(\sqrt{\eta})$ at the fluid–solid interface, and the adaptive grid automatically refines in this layer in exactly the same way that it refines around a physical boundary layer, since the wavelet coefficients of the penalized velocity field are large there.

5.2 Consistency between Penalization Layer and Wavelet Resolution

For the penalization error and the wavelet truncation error to be balanced, the finest wavelet level J is chosen so that the corresponding grid spacing $h_J = 2^{-J}$ resolves the penalization layer thickness, i.e. $h_J = O(\sqrt{\eta})$. This condition links the two independent small parameters of the method — the penalization parameter η and the wavelet threshold ε — and is used in Section 8 to obtain an overall error estimate for the coupled scheme.

5.3 Boundary Data on the Adaptive Grid

Dirichlet data on $\partial\tilde{\Omega}$ (the outer computational box) are imposed directly at the corresponding collocation points. Because interpolating wavelets are interpolatory, imposing a boundary value at a grid point does not disturb the values or derivatives associated with neighbouring points, which allows boundary conditions to be applied without modification of the interior

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stencils, in contrast to finite-difference discretizations near irregular boundaries, which typically require special one-sided or ghost-point stencils.

6. Numerical Algorithm

The complete adaptive wavelet collocation algorithm for one time step, advancing the discrete solution from t^n to $t^{n+1} = t^n + \Delta t$, is summarized as follows:

Given the velocity field u^n on the current adaptive grid X^n, evaluate the mask function $\chi\Omega^s$ and penalization term at all active points.

Compute the forward wavelet transform of u^n; threshold at level ε to obtain the significant coefficient set and extend the grid to X^{n+1} following the procedure of Section 4.2.

Interpolate u^n onto X^{n+1} using the inverse wavelet transform.

Evaluate the convective term $(u\cdot\nabla)u$ and the diffusive term $\nu\nabla^2 u$ on X^{n+1} using the wavelet-derivative operators of Section 4.3.

Advance the momentum equation, including the penalization term, by one Runge–Kutta / Crank–Nicolson sub-step to obtain an intermediate velocity field u^*.

Solve the pressure-Poisson equation $\nabla^2 p = (\rho/\Delta t)\, \nabla\cdot u^*$ on X^{n+1} to obtain p^{n+1}, using a wavelet-preconditioned Krylov iteration.

Correct the intermediate velocity, $u^{n+1} = u^* - (\Delta t/\rho)\nabla p^{n+1}$, so that $\nabla\cdot u^{n+1} = 0$ to within solver tolerance.

Advance $t \leftarrow t + \Delta t$ and return to Step 1.

The dominant computational cost per time step is O(N_points log N_points) for the wavelet transforms and O(N_points) for derivative evaluation, where N_points is the (typically small, adaptively determined) number of active collocation points, in contrast to the O(N_uniform) cost that a uniform grid resolving the same finest scale would require.

7. Representative Benchmark Configurations

The behaviour of the method is illustrated using two standard viscous-flow benchmarks that combine irregular or singular geometric features with well-documented reference solutions in the literature, allowing the compression and refinement properties of the wavelet grid to be assessed against known flow structure.

7.1 Lid-Driven Cavity with a Re-Entrant Corner

A square cavity with a single re-entrant (non-convex) corner is driven by a tangential velocity along the top boundary. The corner singularity produces a local, algebraically decaying pressure field, and the primary and secondary corner eddies span a wide range of length scales. This configuration tests the ability of the adaptive grid to refine locally around a geometric singularity without global mesh refinement.

7.2 Flow Past a Circular Obstacle in a Channel

A circular obstacle embedded within a channel, represented through the volume-penalization mask of Section 5, is used to assess boundary-layer and wake resolution at moderate Reynolds number, where the flow develops a separated shear layer and, beyond the critical Reynolds number, a periodically shedding von Kármán wake. This case exercises both the curved-boundary penalization layer and the time-dependent adaptivity of the grid as the wake structure convects downstream.

7.3 Expected Grid Compression and Convergence Behaviour

For both configurations, the theory of Section 8 predicts that the number of active collocation points required to reach a target error δ scales approximately as $N_{points} \propto \delta^{-d/N}$ rather than the $N_{uniform} \propto \delta^{-d/N}\cdot 2^{((J-j_0))}$ points required by a uniform grid resolving the same finest scale, since the wavelet grid concentrates points only in the boundary-layer and wake regions where the solution gradient is large. The qualitative pattern of local point density with resolution level obtained from the multi-resolution decomposition of the two benchmark flows is summarized in Table 1.

Resolution level j	Grid spacing $h_j = 2^{-j}$	Active points — cavity corner	Active points — cylinder wake
j_0 (coarse)	2^{-4}	all points	all points
$j_0 + 2$	2^{-6}	≈ 35% of level	≈ 40% of level
$j_0 + 4$	2^{-8}	≈ 12% of level	≈ 18% of level
J (finest)	2^{-12}	< 3% of level	< 5% of level

Table 1. Qualitative pattern of active-point fraction retained at each resolution level after thresholding, illustrating the concentration of fine-level points near the re-entrant corner and within the boundary layer / wake, consistent with the compression estimates of Section 8. Precise percentages depend on the Reynolds number, threshold ε, and vanishing-moment order N and are determined by direct simulation.

8. Error and Convergence Analysis

Three sources of error enter the coupled adaptive wavelet / volume-penalization scheme: (i) the wavelet truncation error associated with the threshold ε, (ii) the interpolation error of the underlying scaling function of exactness order N, and (iii) the penalization error associated with η.

8.1 Wavelet Truncation Error

If u ∈ H^s(Ω) for some Sobolev regularity index s ≤ N, standard multiresolution approximation theory gives, for the wavelet interpolant u_J retaining coefficients above threshold ε,

$$\|u - u\_J\|\_L2 \leq C_1\, \varepsilon + C_2\, 2^{-sJ},$$

so that, provided ε is chosen proportional to 2^{-sJ}, the two contributions balance and the overall truncation error converges at the same algebraic rate as a uniform grid at the finest level J, while the number of retained points N_{points} is, by the compression argument of Section 7.3, orders of magnitude smaller than the corresponding uniform-grid count $N_{uniform}$ whenever the solution possesses localized fine-scale structure.

8.2 Penalization Error

Established convergence theory for the Brinkman volume-penalization method gives an L^2-error bound of order √η between the penalized solution uη and the exact no-slip solution u on Ω,

$$\|u - u\eta\|\_L2(\Omega) \leq C_3\, \sqrt{\eta},$$

and, correspondingly, an O(η) error in derived quantities away from the immediate vicinity of the boundary. Balancing this contribution against the wavelet truncation error of Section 8.1 requires η = O(2–^{2}sJ), which together with the resolution condition of Section 5.2 determines the finest level J needed for a target overall accuracy.

8.3 Time-Discretization Error

The semi-implicit projection scheme of Section 4.4 is first-order accurate in time for the pressure and second-order accurate for the velocity when Crank–Nicolson treatment of the viscous term is combined with a second-order Runge–Kutta treatment of the convective term; the time step Δt is restricted by a convective (CFL) condition Δt ≤ C·h_J/‖u‖_∞ on the finest active grid spacing, rather than by the diffusive limit, since the viscous term is treated implicitly.

8.4 Combined Estimate

Collecting the three contributions, the total error of the adaptive wavelet collocation solution with volume-penalized boundaries satisfies

$$\|u - u\_h\|\_L2 \leq C_1\varepsilon + C_2\, 2^{-sJ}) + C_3\, \sqrt{\eta} + C_4\, \Delta t^p,$$

with p ∈ {1,2} depending on the quantity of interest, so that a consistent refinement strategy — simultaneously decreasing ε, increasing J, decreasing η as 2^{-2}sJ, and decreasing Δt proportionally to h_J — yields overall convergence at the rate dictated by the solution regularity s and the vanishing-moment order N of the wavelet basis, while the adaptive point count grows far more slowly than for a uniform discretization at the same finest level.

9. Discussion

The framework developed here shows that the two central difficulties of simulating viscous flow in irregular domains — resolving multiscale flow structure and enforcing boundary conditions on non-Cartesian geometry — can be addressed within a single, mathematically consistent multiresolution formulation. The interpolating-wavelet collocation grid supplies adaptivity and a controllable, theory-backed error bound, while volume penalization removes the need for body-fitted meshing at the cost of an additional, but independently controllable, small parameter η. The coupling condition $h_J = O(\sqrt{\eta})$ linking the finest wavelet level to the penalization parameter is the key structural relation that keeps the two sources of error balanced, and follows directly from combining classical penalization theory with multi-resolution approximation theory.

A practical limitation of the approach is that the pressure-Poisson solve on an irregular, dynamically changing point set requires an iterative solver whose preconditioning must itself be wavelet-adapted to retain the near-linear computational scaling of the transform and derivative steps; a poorly preconditioned solve can dominate the overall cost and erode the efficiency gained from grid adaptively. Extension of the analysis to three-dimensional, high-Reynolds-number turbulent flow, where the range of active scales is substantially larger, is a natural direction for further theoretical and computational development.

10. Conclusion and Future Work

Combining interpolating wavelets for spatial adaptively with a volume-penalization formulation for boundary enforcement, this research has introduced an adaptive wavelet collocation framework for the incompressible Navier-Stokes equations in irregular geometries, which is based on multi-resolution analysis. An error analysis that takes into account wavelet truncation, interpolation order, penalization, and time-discretization errors was derived after a review of the mathematical foundations of multi-resolution analysis and interpolating wavelets. The governing equations were discretized on a dynamically adapted collocation grid. Promising avenues for future doctoral research include expanding the framework to three-dimensional turbulent flow, proving the stability of the fully coupled adaptive scheme, and developing wavelet-adapted pre-conditioners for the pressure-Poisson sub problem.

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